CS 30900, Spring 2016 Rules of Boolean Algebra

1. Below we use the symbol $\equiv$ to represent *logical equivalence*. So the $\equiv$ symbol in

$$LHS \equiv RHS$$

means that $LHS \leftrightarrow RHS$ is a tautology, which is the same thing as

$$(LHS \leftrightarrow RHS) \leftrightarrow T$$

where $\leftrightarrow$ has its usual meaning as the boolean binary equality operator (that is, the negation of xor), and T represents the boolean operator that is always true.

   1. $p \lor q \equiv q \lor p$ Commutative
   2. $p \land q \equiv q \land p$ Commutative
   3. $p \lor (q \lor r) \equiv (p \lor q) \lor r$ Associative
   4. $p \land (q \land r) \equiv (p \land q) \land r$ Associative
   5. $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$ Distributive
   6. $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$ Distributive
   7. $\neg(p \lor q) \equiv \neg p \land \neg q$ DeMorgan's
   8. $\neg(p \land q) \equiv \neg p \lor \neg q$ DeMorgan's
   9. $p \land T \equiv p$ Identity
   10. $p \lor F \equiv p$ Identity
   11. $p \land p \equiv p$ Idempotency
   12. $p \lor p \equiv p$ Idempotency
   13. $p \land F \equiv F$ Domination
   14. $p \lor T \equiv T$ Domination
   15. $p \lor \neg p \equiv T$ Excluded Middle
   16. $p \land \neg p \equiv F$ Excluded Middle
   17. $p \land (p \lor q) \equiv p$ Absorption
   18. $p \lor (p \land q) \equiv p$ Absorption
   19. $p \to q \equiv \neg q \to \neg p$ Contrapositive
   20. $p \to q \equiv \neg p \lor q$ Law of implication
   21. $\neg(p \to q) \equiv p \land \neg q$
   22. $p \leftrightarrow q \equiv (p \to q) \land (q \to p)$ Biconditional
   23. $p \leftrightarrow q \equiv \neg(p \text{ xor } q)$ Inclusive And
   24. $\neg(p \leftrightarrow q) \equiv (p \land \neg q) \lor (\neg p \land q)$ Exclusive Or
   25. $\neg(p \leftrightarrow q) \equiv (p \lor q) \land \neg(p \land q)$ Exclusive Or
   26. $p \to (q \land r) \equiv (p \to q) \land (p \to r)$
   27. $p \to (q \lor r) \equiv (p \to q) \lor (p \to r)$
   28. $(p \land q) \to r \equiv (p \land \neg r) \to \neg q$
   29. $(p \to q) \land (q \to r) \land (r \to p) \equiv (p \leftrightarrow q) \land (q \leftrightarrow r) \land (p \leftrightarrow r)$