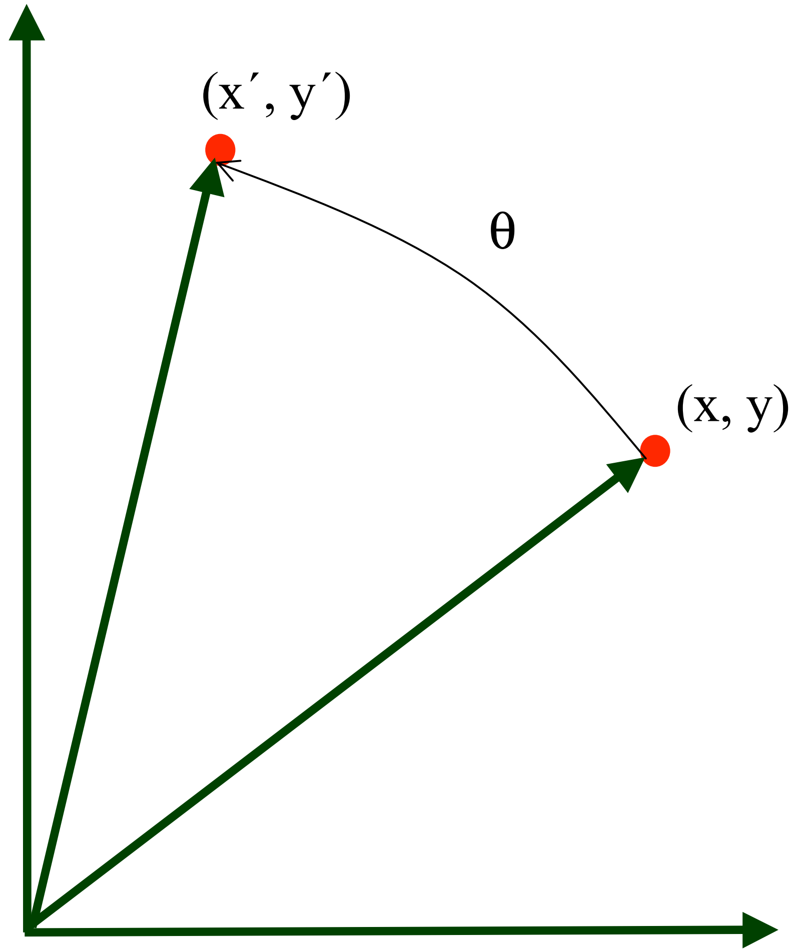
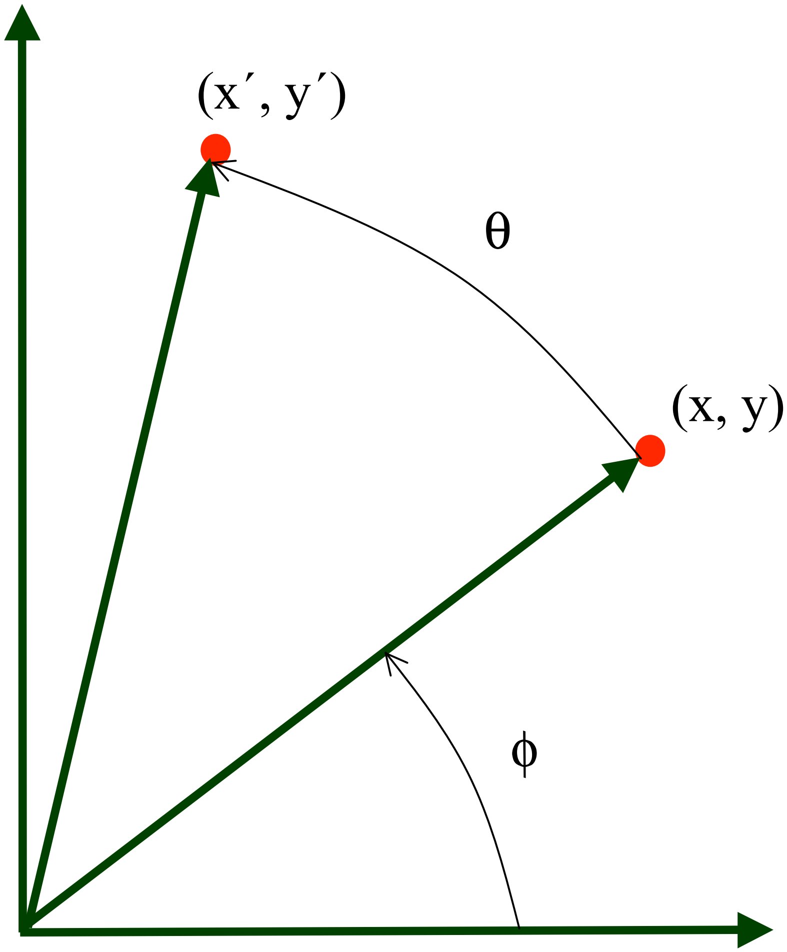
Rotation 2D



$$\begin{cases} x' = x\cos\theta - y\sin\theta \\ y' = x\sin\theta + y\cos\theta \end{cases}$$

Rotating a point P=(x,y) through an angle θ about the origin O(0,0) counterclockwise means to determine another point Q=(x′,y′) on the circle centred at O such that θ=∠POQ.

Rotation 2D: equations



$$\begin{cases} x = r\cos\phi \\ y = r\sin\phi \end{cases}$$

$$\begin{cases} x' = r\cos(\phi+\theta) \\ y' = r\sin(\phi+\theta) \end{cases}$$

Expanding the expressions of x´ and y´, we have:

$$\begin{cases} x' = r\cos\phi\cos\theta - r\sin\phi\sin\theta \\ y' = r\cos\phi\sin\theta + r\sin\phi\cos\theta \end{cases}$$

Replacing r cos(f) and r sin(f) by x and y in the previous equations, we get:

$$\begin{cases} x' = x\cos\theta - y\sin\theta \\ y' = x\sin\theta + y\cos\theta \end{cases}$$

Rotation 2D: matrix representation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Although sin(θ) and cos(θ) are not linear functions of θ,
  - x’ is a linear combination of x and y
  - y’ is a linear combination of x and y