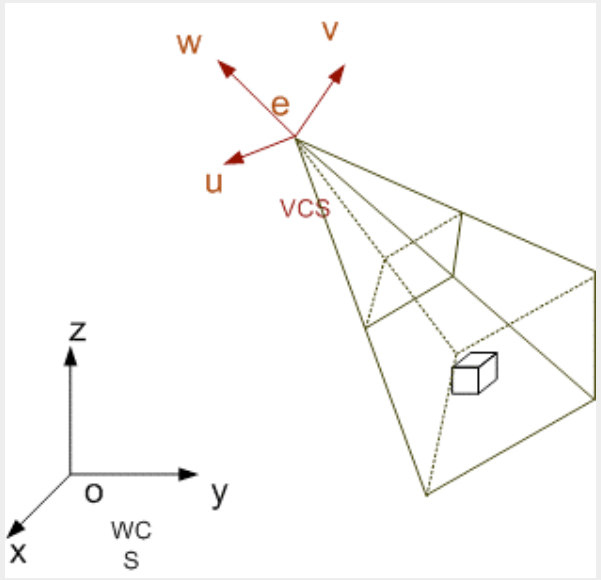
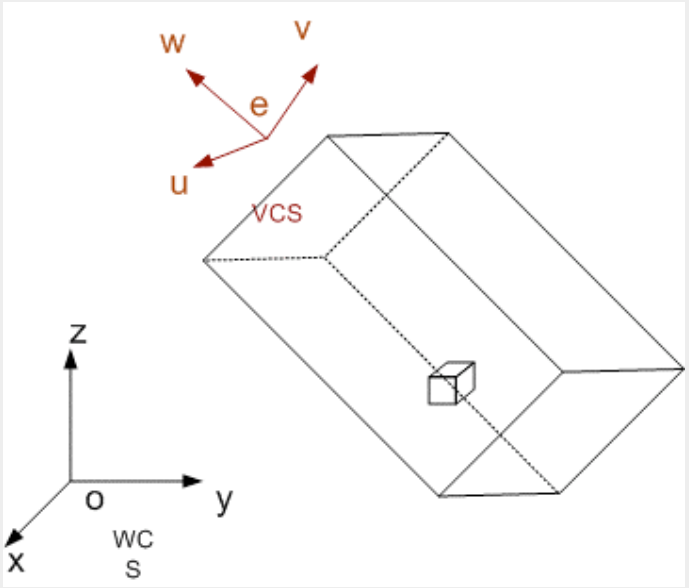
View Projections

- Transform from camera space to normalized view space
- Two basic kinds:
  - *Perspective projection*: make things farther away seem smaller
    - Most common for computer graphics
    - Simple model of human eye, or camera lens
    - (Actually, a model of an ideal *pinhole camera*)
  - *Orthographic projection*: simply flatten, without any perspective
    - Used for architectural or plan views (top,side,front)
    - Not used for realistic rendering
- Others, more complex:
  - lens, with focus & depth of field
  - fish-eye lens
  - dome projection
  - computations don't easily fit into basic hardware rendering pipeline

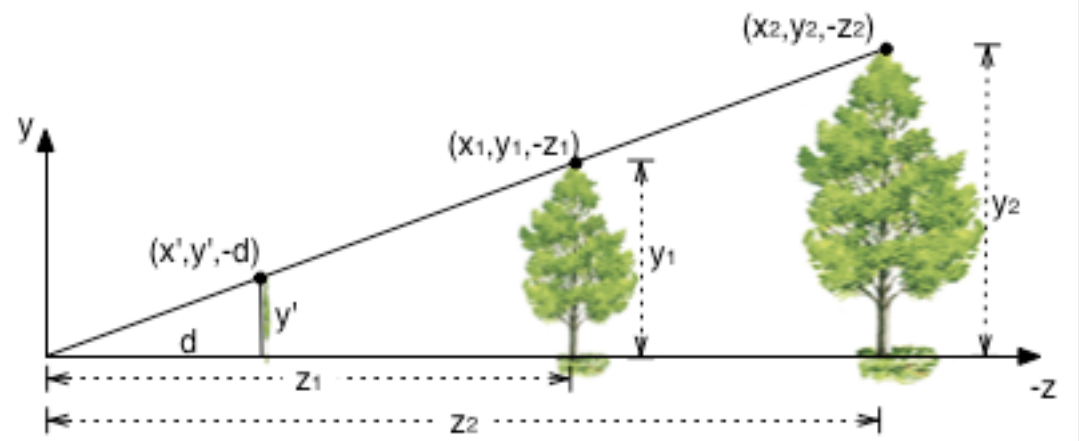
View Volume

- A 3D shape in world space that represents the volume viewable by the camera





Perspective Projection



- Assume that we have "film" at distance *d* from the eye
- Distant tall object projects to same height as near small object
- By similar triangles, we have:

$$\frac{y'}{d} = \frac{y_1}{z_1} = \frac{y_2}{z_2}$$

Giving the transformation relations:

$$y' = d\frac{y}{z}, \qquad x' = d\frac{x}{z}$$

- Notice: divide by z
  - not a linear operation!

Perspective Projection

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} d_1 \frac{x}{z} \\ d_2 \frac{y}{z} \\ A + \frac{B}{z} \end{bmatrix}$$

- Not a linear equation
  - not an affine transformation
  - doesn't preserve angles-but *does* preserve straight lines
  - Note: it will blow up if z=0 (object at the eye)
- Z maps to *pseudo-distance*
  - necessary to preserve straight lines
  - maintains depth order when B<0: if $z_1<z_2$ then $z'_1<z'_2$
- We'll come up with values for d_1, d_2, A, and B, in a little while
  - will choose them to keep area of interest within -1 to 1 in x,y,z
- Ugly formula. Make it work with homogeneous matrices…

Homogeneous Perspective Projection

- The homogeneous perspective projection matrix. Notice the last row!

$$\mathbf{P} = \begin{bmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & A & B \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

- Multiply it by a homogeneous point

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \mathbf{P} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} d_1 x + 0 + 0 + 0 \\ 0 + d_2 y + 0 + 0 \\ 0 + 0 + Az + B \\ 0 + 0 + z + 0 \end{bmatrix} = \begin{bmatrix} d_1 x \\ d_2 y \\ Az + B \\ z \end{bmatrix}$$

- Notice that the result doesn't have *w*=1. So *divide by w*:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} \Rightarrow \begin{bmatrix} x'/w' \\ y'/w' \\ z'/w' \\ w'/w' \end{bmatrix} = \begin{bmatrix} d_1 x / z \\ d_2 x / z \\ A + B / z \\ 1 \end{bmatrix}$$

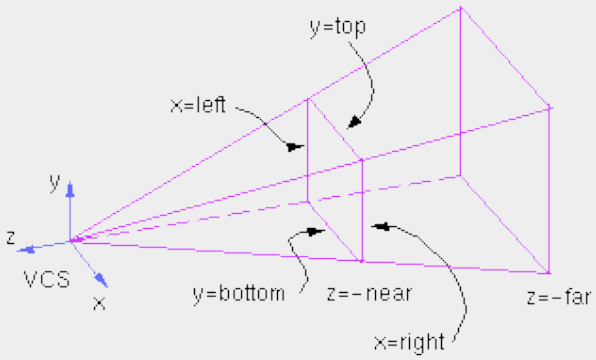
Homogeneous Perspective Transform

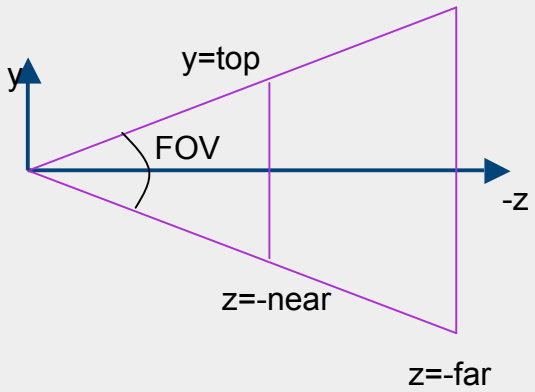
- As always, there's some deep math behind this…
  - 3D projective space
- For practical purposes:
  - Use homogeneous matrices normally
  - Modeling & viewing transformations use *affine matrices*
    - points keep w=1
    - no need to divide by w when doing modeling operations or transforming into camera space
  - Projection transform uses *perspective matrices*
    - w not always 1
    - divide by w after performing projection transform
    - AKA *perspective divide*, *homogeneous divide*
  - GPU hardware does this

Perspective view volume

- A perspective camera with a rectangular image describes a pyramid in space
  - The tip of the pyramid is at the eye point
  - The pyramid projects outward in front of the camera into space
  - Nominally the pyramid starts at the eye point and goes out infinitely…
  - But, to avoid divide-by-zero problems for objects close to the camera
    - introduce a *near clipping plane*
    - objects closer than that are not shown
    - chops off the tip of the pyramid
  - Also, to avoid floating-point precision problems in the Z buffer
    - introduce a *far clipping plane*
    - objects beyond that are not shown
    - defines the bottom of the pyramid
  - A pyramid with the tip cut off is a truncated pyramid, AKA a *frustum*
  - The standard perspective view volume is called the *view frustum*

View Frustum





Parameterized by:

- left,right,top,bottom (generally symmetric)
- near,far

Or, when symmetric, by:

- *Field of view* (FOV), *aspect ratio*
- near,far
- Aspect ratio is the x/y ratio of the final displayed image. Common values:
  - 4/3 for TV & old movies; 1.66 for cartoons & European movies; 16/9 for American movies & HDTV; 2.35 for epic movies

$$\text{aspect ratio} = \frac{right - left}{top - bottom} = \frac{right}{top}$$

$$\tan(FOV / 2) = \frac{top}{near}$$

Frustum Projection Matrix

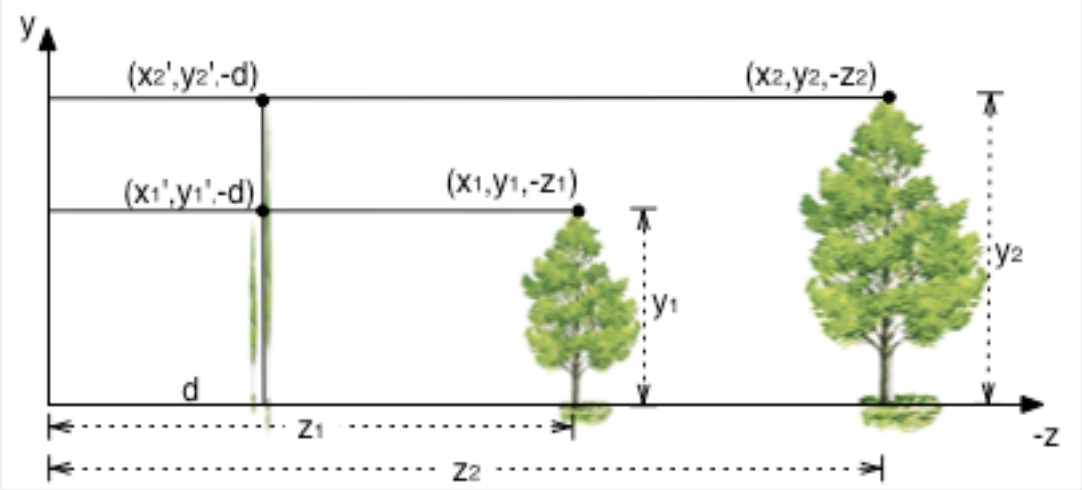
- We can think of the view frustrum as a distorted cube, since it has six faces, each with 4 sides
- The perspective projection warps this to a cube.
  - Everything inside gets distorted accordingly
  - By setting the parameters properly, we get the cube to range from -1 to 1 in all dimensions: i.e., normalized view space

$$\mathbf{P}_{persp}(FOV, aspect, near, far) = \begin{bmatrix} \frac{1}{aspect \cdot \tan(FOV/2)} & 0 & 0 & 0 \\ 0 & \frac{1}{\tan(FOV/2)} & 0 & 0 \\ 0 & 0 & \frac{near+far}{near-far} & \frac{2 \cdot near \cdot far}{near-far} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Perspective Camera Class

```
class Perspective(Camera) {
   Vector3 Eye;
   Vector3 Target;
   float FOV,Aspect,NearClip,FarClip;
   Matrix getProjection();
};
```

Orthographic projection



- Simple: x'=x, y'=y, z'=z
- For scaling purposes, we'll introduce d_1, d_2, A, and B
  - Again, will choose them so that region of interest is in -1 to 1

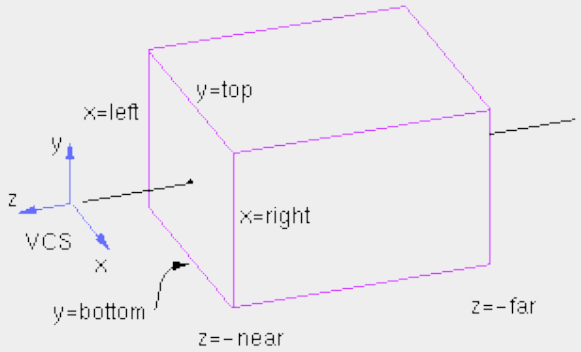
Homogeneous Orthographic Projection

- The orthographic projection matrix is:

$$\mathbf{P} = \begin{bmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & A & B \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- This is an ordinary affine transform (scale+translate)
  - when transforming a point, keeps w=1
  - no need to divide by w
  - but… no harm done dividing by w
- Send the GPU either an orthonormal or perspective projection matrix
  - it can divide by w in either case
  - don't need to special-case the two types of projections

Orthographic View Volume



- Parametrized by: right, left, top, bottom, near, far
- Or, if symmetric, by: width, height, near, far

Orthographic Projection Matrix

- Simply translates and scales to transform the view volume to -1 to 1 in all dimensions: normalized view coordinates.

$$\mathbf{P}_{ortho}(right,left,top,bottom,near,far) = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bottom} & 0 & -\frac{top+bottom}{top-bottom} \\ 0 & 0 & \frac{2}{far-near} & \frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{P}_{ortho}(width,height,near,far) = \begin{bmatrix} \frac{2}{width} & 0 & 0 & 0 \\ 0 & \frac{2}{height} & 0 & 0 \\ 0 & 0 & \frac{2}{far-near} & \frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$