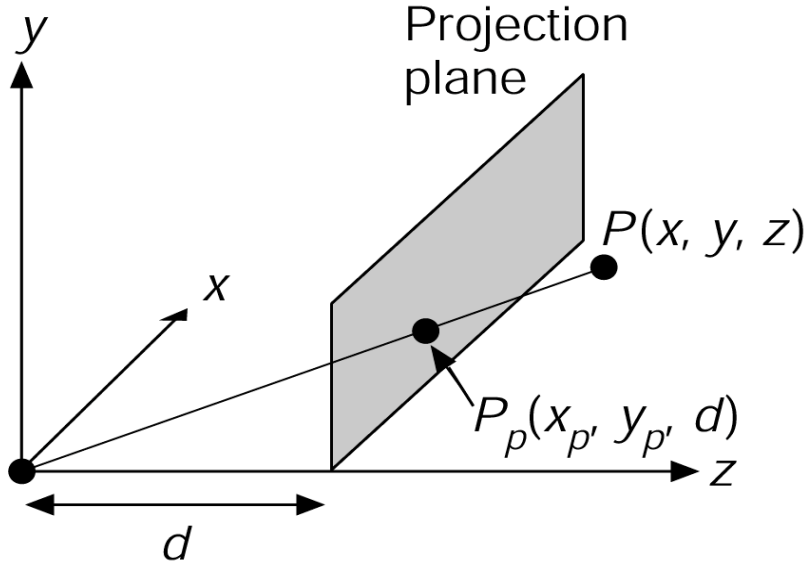
The Perspective Projection



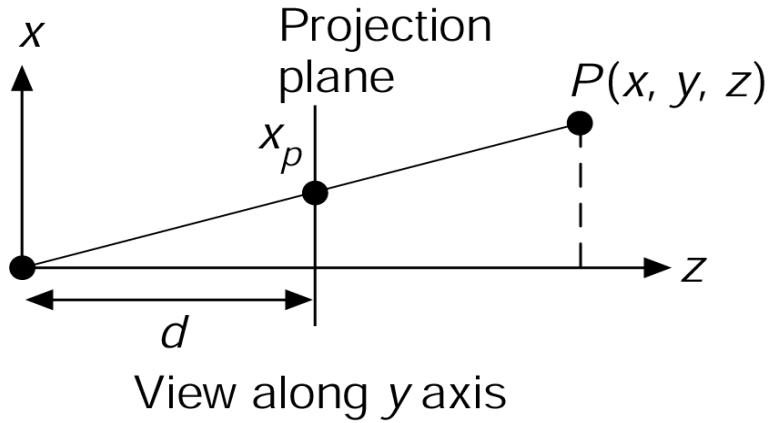
Determining scale

- Consider
  - Point P
  - Projected onto projection plane as point P_p
- Idea: compute ratios via similar triangles

The Perspective Projection

- In the x direction ratio is

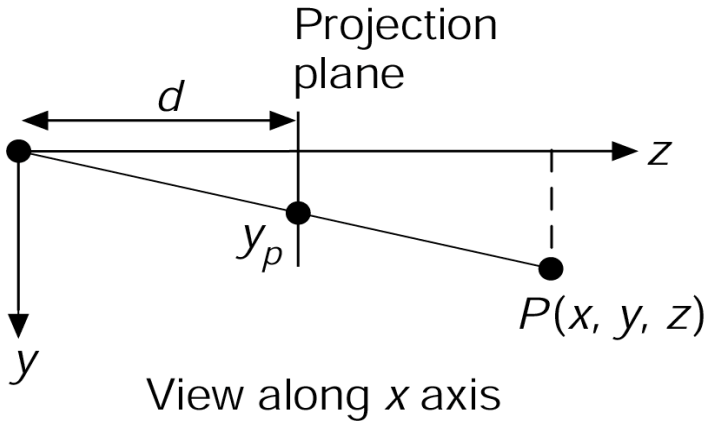
$$\frac{z}{d} = \frac{x}{x_p} \qquad x_p = \frac{x}{z/d}$$



The Perspective Projection

- In the y direction ratio is

$$\frac{z}{d} = \frac{y}{y_p} \qquad y_p = \frac{y}{z/d}$$



The Perspective Projection

- Homogenous perspective projection matrix

$$M_{\text{per}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$

Assumes VPN is z axis.

The Perspective Projection

- Homogenous perspective projection

$$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

The Perspective Projection

- Homogenous perspective projection to 3D

$$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ d \end{bmatrix}$$