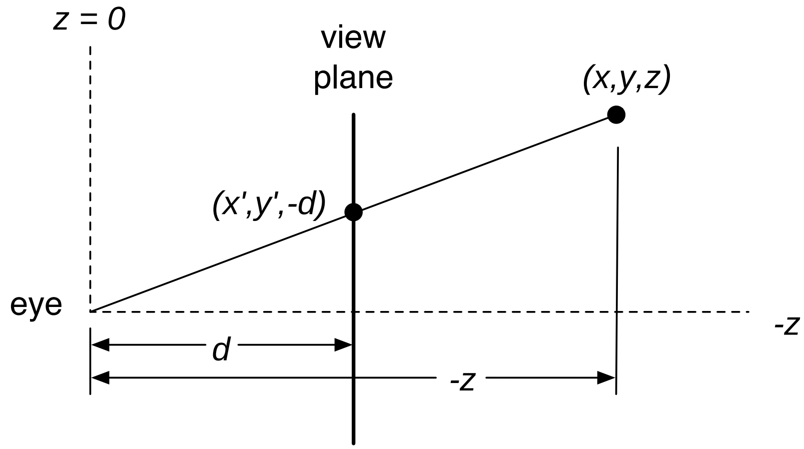
Perspective Projection onto View Plane



Via similar triangles (note $z < 0$):

$$\begin{aligned} \frac{x'}{d} &= \frac{x}{-z} \\ \frac{y'}{d} &= \frac{y}{-z} \\ z' &= -d \end{aligned}$$

$$(x', y', z') = \left(\frac{xd}{-z}, \frac{yd}{-z}, -d\right).$$

Perspective Transformation

- Our perspective transformation is <u>not</u> linear (we can <u>not</u> describe it with a (simple) matrix).
- We exploit the use of homogeneous coordinates

$$(x, y, z, w) \equiv (x/w, y/w, z/w, 1) \quad (w \neq 0).$$

- We then create the following perspective matrix

$$\begin{bmatrix} xd \\ yd \\ zd \\ -z \end{bmatrix} = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & d & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

  After transforming our point, we perform a homogeneous **perspective division**:

$$(xd, yd, zd, -z) \mapsto \left(\frac{-xd}{z}, \frac{-yd}{z}, -d, 1 \right)$$