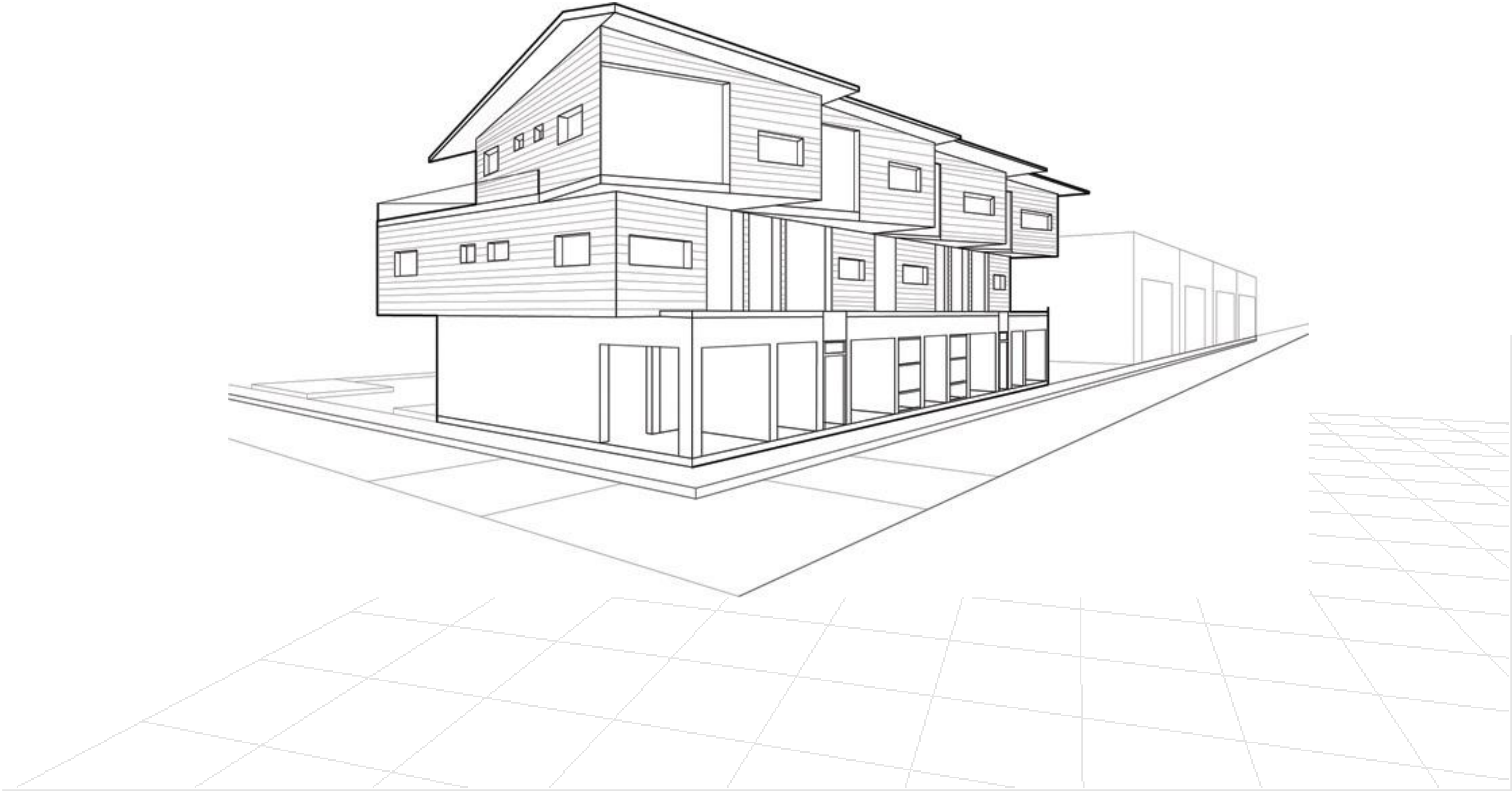
Viewing in 3D

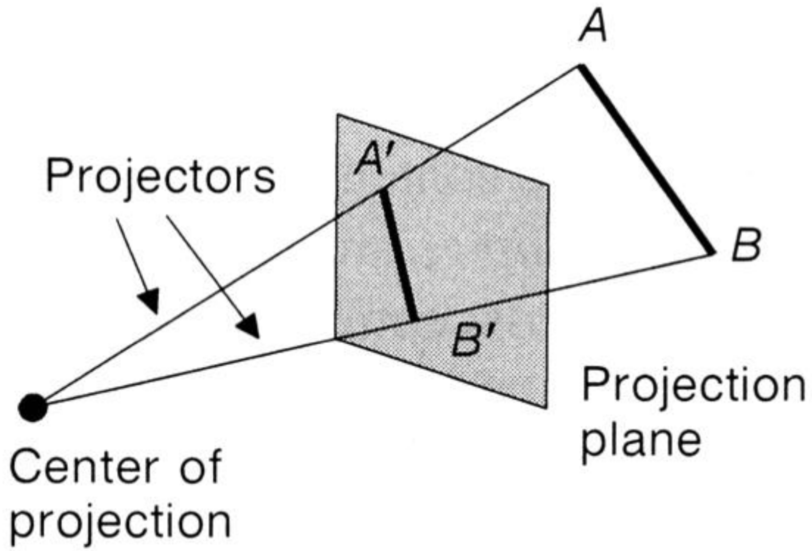


Projections

- Display device (a screen) is 2D…
  - How do we map 3D objects to 2D space?
- 2D to 2D is straight forward…
  - 2D window to world.. and a viewport on the 2D surface.
  - Clip what won't be shown in the 2D window, and map the remainder to the viewport.
- 3D to 2D is more complicated…
  - Solution : Transform 3D objects on to a 2D plane using ***projections***

Projections

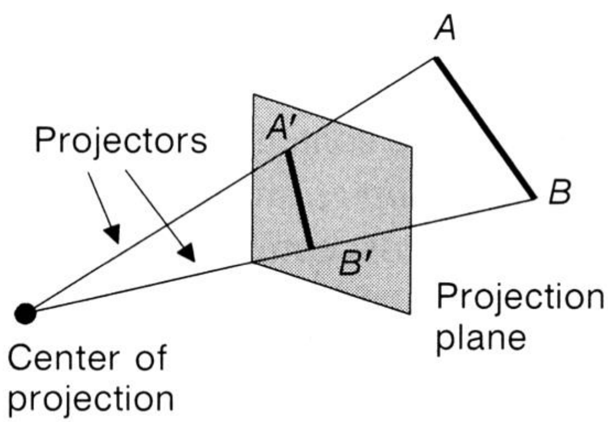
- Projections: key terms…
  - *Projection* from 3D to 2D is defined by straight *projection rays* (*projectors*) emanating from the '*center of projection*', passing through each point of the object, and intersecting the '*projection plane*' to form a projection.

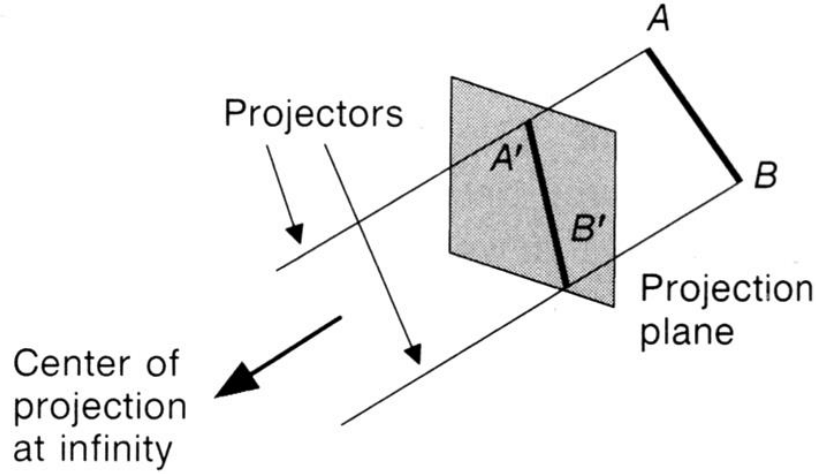


Types of projections

- 2 types of projections
  - *perspective* and *parallel*.

- Key factor is the *center of projection*.
  - if distance to center of projection is finite : perspective
  - if infinite : parallel





Perspective v Parallel

- Perspective:
  - visual effect is similar to human visual system...
  - has 'perspective foreshortening'
    - size of object varies inversely with distance from the center of projection.
  - angles only remain intact for faces parallel to projection plane.
- Parallel:
  - less realistic view because of no foreshortening
  - however, parallel lines remain parallel.
  - angles only remain intact for faces parallel to projection plane.

Perspective Projections

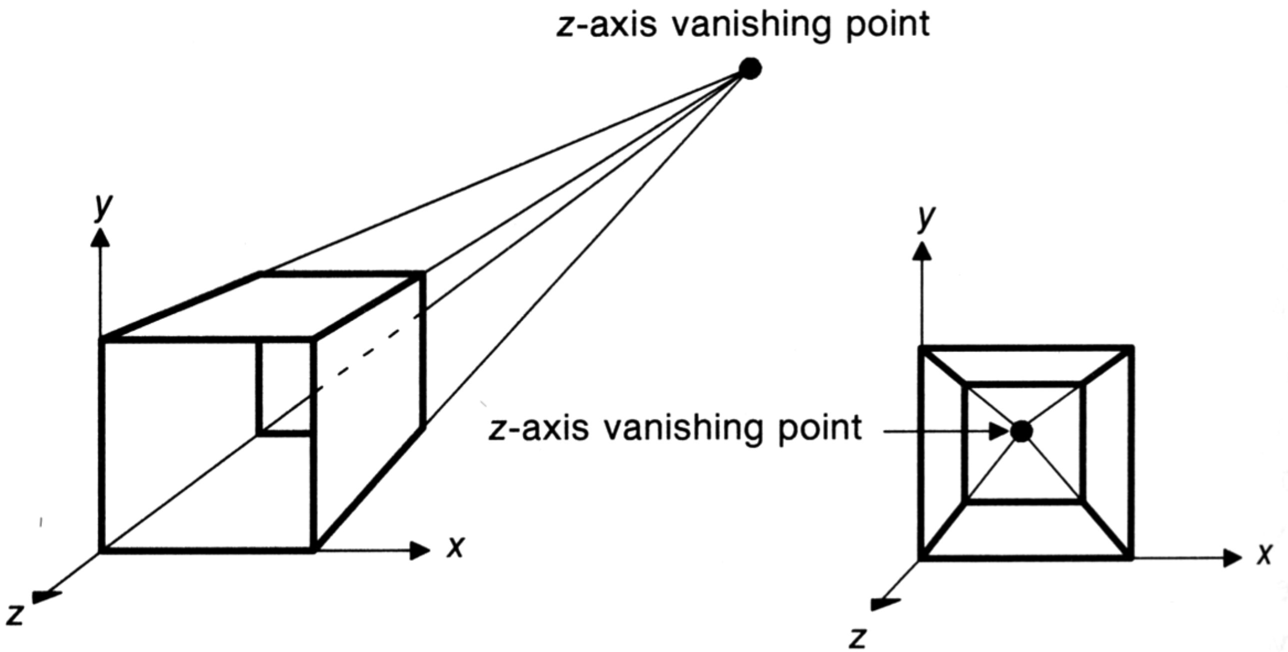
- Any parallel lines *not* parallel to the projection plane, converge at a vanishing point.
  - There are an infinite number of these, 1 for each of the infinite amount of directions line can be oriented.

- If a set of lines are parallel to one of the three principle axes, the vanishing point is called an *axis vanishing point*.
  - There are at most 3 such points, corresponding to the number of axes cut by the projection plane.

Perspective Projections

- 2 different examples of a one-point perspective projection of a cube.

  (note: x and y parallel lines do not converge)

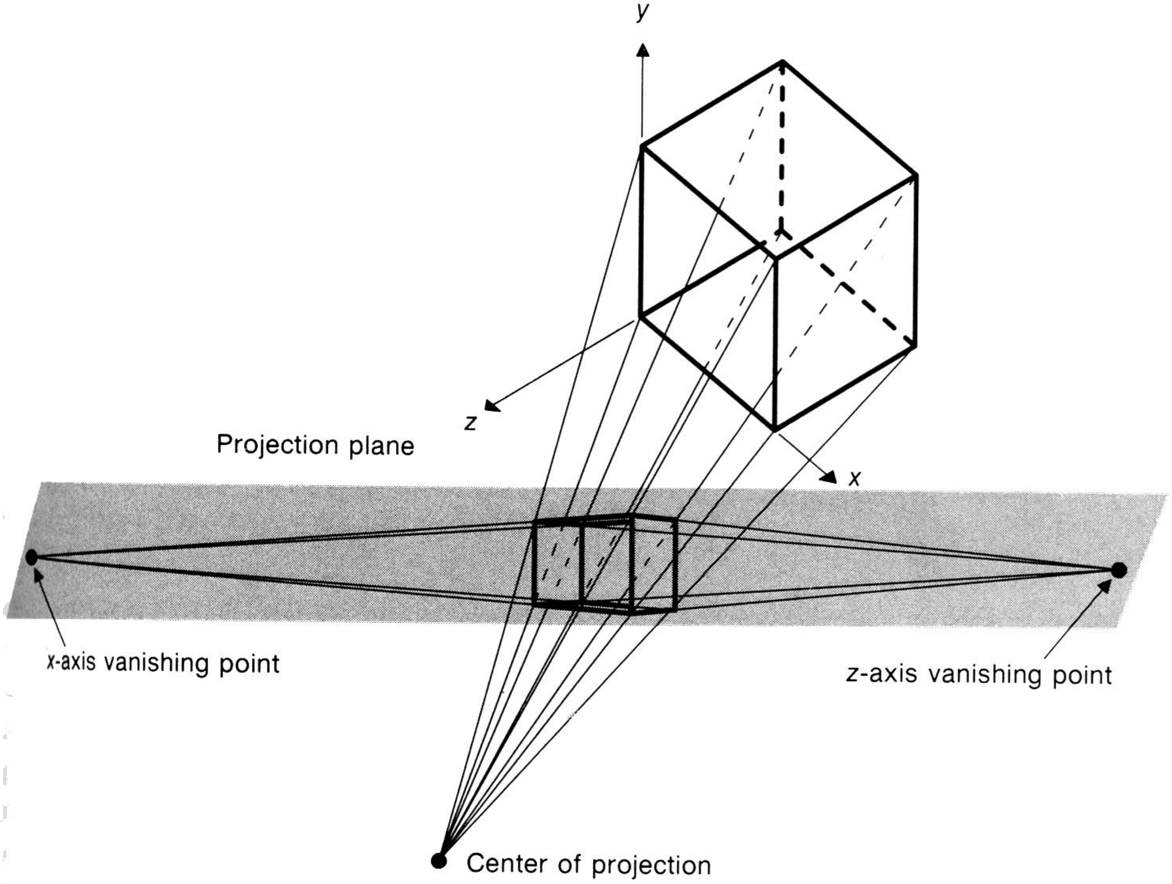


Perspective Projections

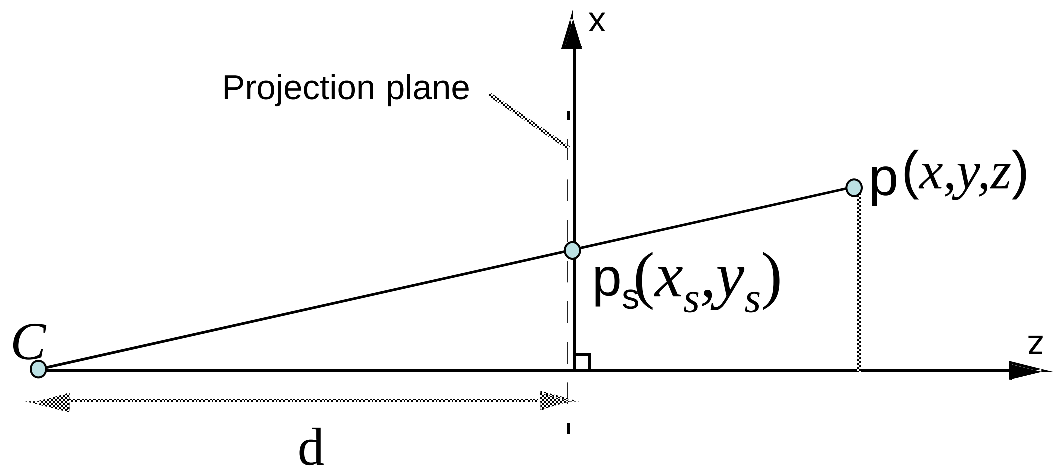
- Two-point perspective projection:
  - This is often used in architectural, engineering and industrial design drawings.
  - Three-point is used less frequently as it adds little extra realism to that offered by two-point perspective projection.

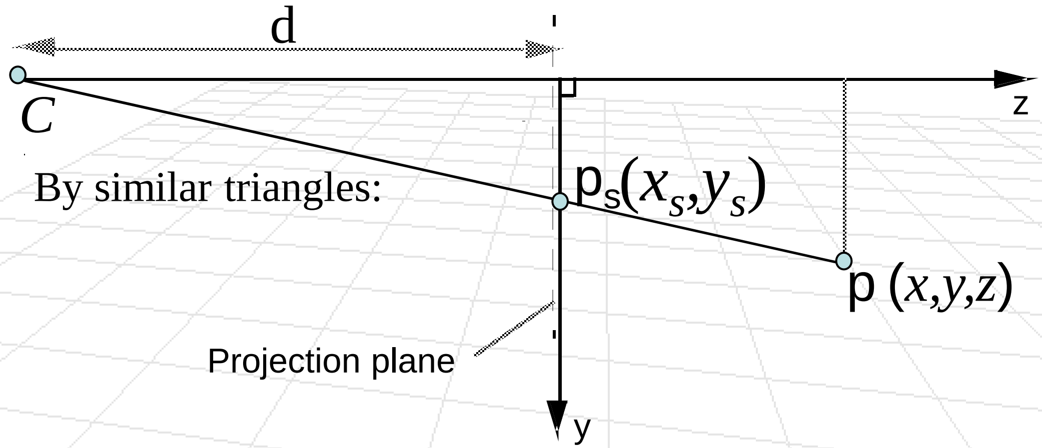
Perspective Projections

- Two-point perspective projection:



Perspective Projections





By similar triangles:

$$\frac{x}{z+d} = \frac{x_s}{d}$$

$$x_s = \frac{x}{1+\frac{z}{d}}$$

$$y_s = \frac{y}{1+\frac{z}{d}}$$

Perspective Projections

$$\begin{pmatrix} x_s \\ y_s \\ 0 \\ 1+\dfrac{z}{d} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x_s \\ y_s \\ 0 \\ 1+\dfrac{z}{d} \end{pmatrix} = \begin{pmatrix} \dfrac{x_s}{1+\dfrac{z}{d}} \\ \dfrac{y_s}{1+\dfrac{z}{d}} \\ 0 \\ 1 \end{pmatrix}$$

Parallel Projections

- 2 principle types:
  - *orthographic* and *oblique*.
- Orthographic :
  - direction of projection = normal to the projection plane.
- Oblique :
  - direction of projection != normal to the projection plane.

Parallel Projections

- Orthographic (or orthogonal) projections:
  - front elevation, top-elevation and side-elevation.
  - all have projection plane perpendicular to a principle axes.

- Useful because angle and distance measurements can be made...

- However, As only one face of an object is shown, it can be hard to create a mental image of the object, even when several view are available.

Parallel Projections

- Orthogonal projections: