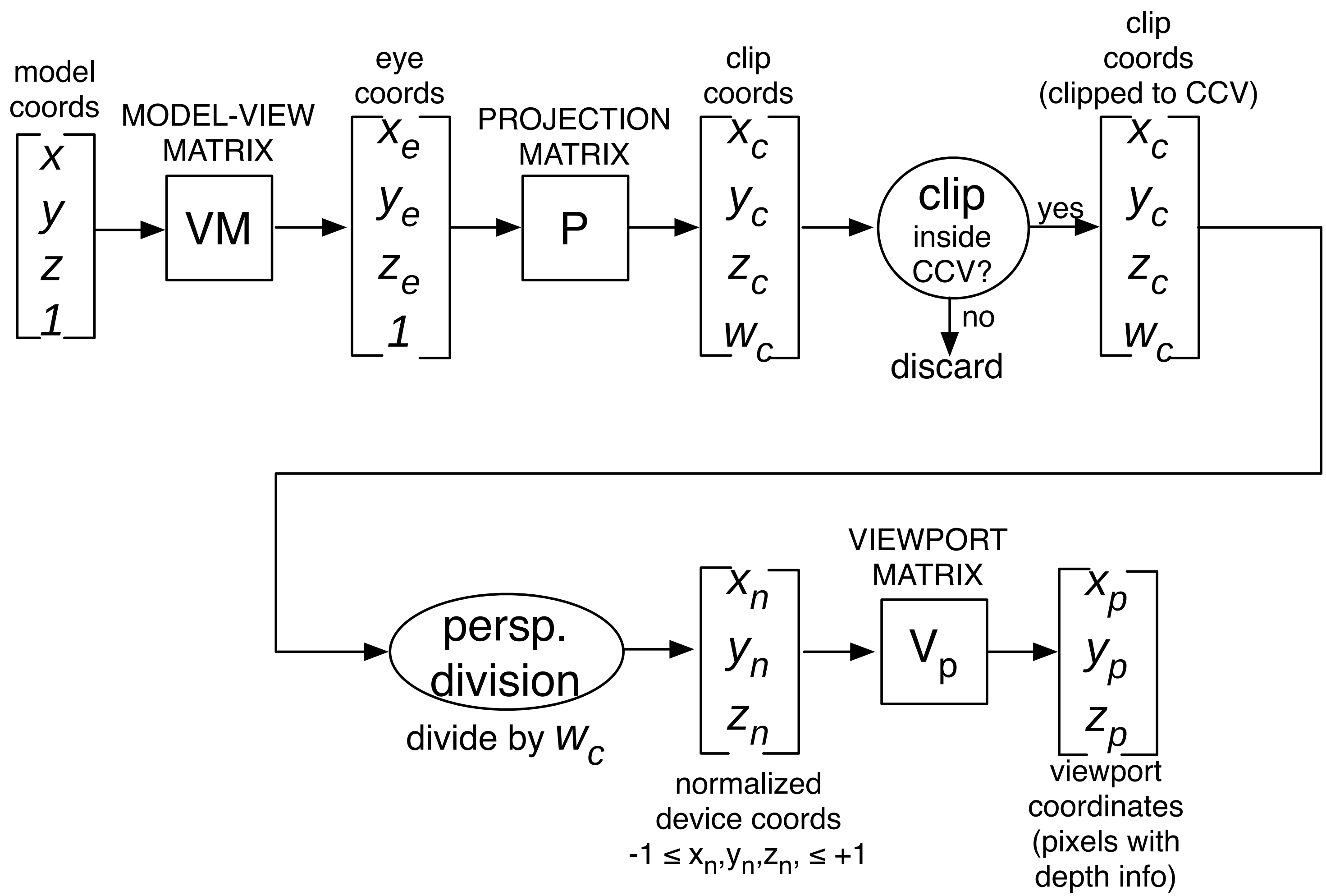
Clipping in 4-D

CS/Cpts 442/452

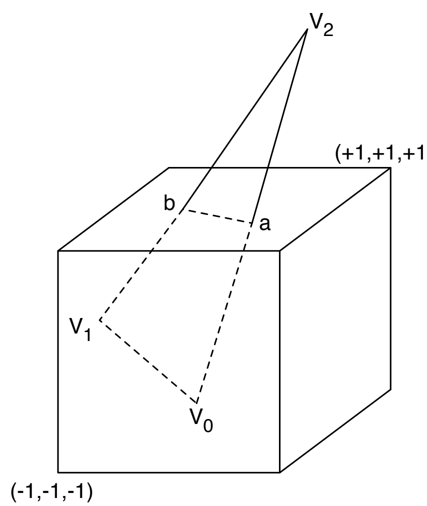
October 3, 2007

A Vertex's Trip Down the Graphics Pipeline



Clipping to the Canonical Clipping Volume (CCV)

- The projection matrix maps points to the CCV (ortho / persp).
- The CCV is conducive for clipping:
  - Faces are parallel to the principle axes;
  - Center at origin with faces at unit distances from origin.
- Below the triangle $\triangle V_0, V_1, V_2$ clipped to a quadrilateral $\square V_0, V_1, b, a$.
  - New vertices a and b (and their associated attributes; e.g., color) are computed via *interpolation.*



Test to see if point inside CCV

- Clipping is performed in homogeneous coordinates.
- (x, y, z, w) is right of the $x = -1$ plane iff:

  $$\frac{x}{w} > -1 \quad \text{or} \quad x > -w \quad \text{or} \quad w + x > 0.$$

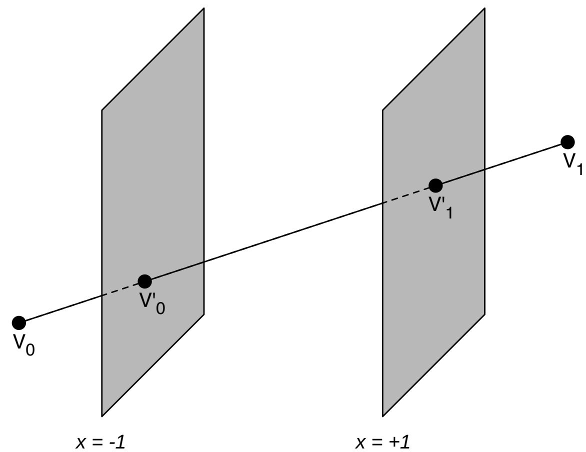
  (x, y, z, w) is left of the $x = +1$ plane iff:

  $$w - x > 0.$$

- For each of the six faces of the cube, we have the following six conditions:

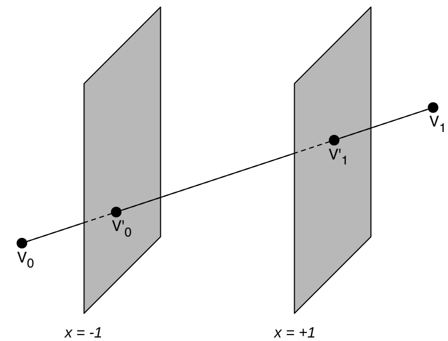
condition	*clip plane*
$w + x > 0$	$x = -1$
$w - x > 0$	$x = +1$
$w + y > 0$	$y = -1$
$w - y > 0$	$y = +1$
$w + z > 0$	$z = -1$
$w - z > 0$	$z = +1$

Clipping a line segment $\overline{V_0V_1}$ with endpoints $V_0 = (x_0, y_0, z_0, w_0)$ and $V_1 = (x_1, y_1, z_1, w_1)$



- **trivial accept**: both V_0 and V_1 inside CCV.
- **trivial reject**: both V_0 and V_1 lie outside the same plane (i.e., both fail on the same condition).
- Otherwise, we find a plane that the line straddles and "trim" the line (Trim $\overline{V_0V_1}$ to $\overline{V_0'V_1}$ in figure).
- Repeat until line accepted or rejected.

Finding the intersection with a CCV plane



- Represent the line segment parametrically:

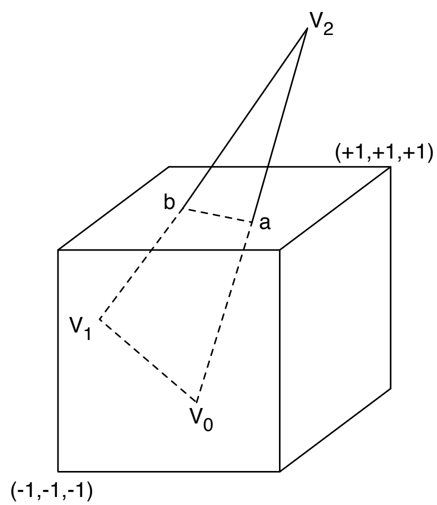
$$\begin{bmatrix} x(t) \\ y(t) \\ z(t) \\ w(t) \end{bmatrix} = t \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ w_0 \end{bmatrix} + (1-t) \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ w_1 \end{bmatrix} \quad t \in [0, 1].$$

- Clipping to $x = -1$ plane we solve the following for t :

$$\frac{x(t)}{w(t)} = \frac{tx_0 + (1-t)x_1}{tw_0 + (1-t)w_1} = -1.$$

- Plug $t = \frac{-w_1 - x_1}{(x_0 - x_1) + (w_0 - w_1)}$ into parametric equation yielding V_0'.

Clipping a Polygon



- The Sutherland-Hodgmen polygon clipping algorithm easily generalizes for CCV clipping.
- OpenGL: convex, (approx) planar polygons.
- Same clipping algo works for both orthographic and perspective projections.
- Attributes (e.g., color) associated with each vertex need to be clipped too.