

1. Projection Transformation (Camera Coordinates to Viewplane Coordinates)

Let (x_c, y_c, z_c) be a point in camera coordinates and let (x_p, y_p, z_p) be its perspective projection onto the viewplane. Then

$$x_p = -x_c/z_c, \quad y_p = -y_c/z_c, \quad z_p = -1.$$

2. Clipping Equations

Let v_0 and v_1 be the endpoints of a line segment in the viewplane $z = -1$. If the line segment crosses the line $x = 1$, or $x = -1$, or $y = 1$, or $y = -1$ (respectively), then we solve for t in the equation

$$(1-t)v_0 + tv_1 = (1, y_c)$$

$$(1-t)v_0 + tv_1 = (-1, y_c)$$

$$(1-t)v_0 + tv_1 = (x_c, 1)$$

$$(1-t)v_0 + tv_1 = (x_c, -1)$$

(respectively) to find the point v_c where the line segment should be clipped. Then use that value of t to interpolate color from v_0 and v_1 to v_c ,

$$(r(t), g(t), b(t)) = (1-t)(r(0), g(0), b(0)) + t(r(1), g(1), b(1)).$$

3. Viewplane to Pixelplane Transformation

Let $(x_p, y_p, -1)$ be a point within the viewplane's view rectangle and let (x_{vp}, y_{vp}) be its transformation to the logical viewport in the renderer's pixelplane. Then

$$x_{vp} = 0.5 + (w/2.001)(x_p + 1),$$

$$y_{vp} = 0.5 + (h/2.001)(y_p + 1).$$

Points (x_{vp}, y_{vp}) in the renderer's logical viewport have coordinates that satisfy

$$0.5 \leq x_{vp} < w + 0.5 \quad \text{and} \quad 0.5 \leq y_{vp} < h + 0.5.$$

4. Pixelplane to Pixel Transformation

Let (x_{vp}, y_{vp}) be a point in the renderer's logical viewport (in the pixelplane) and let (x, y) be the equivalent pixel in the framebuffer's viewport. Then

$$x = (\text{int})\text{Math.round}(x_{vp}) - 1$$

$$y = h - (\text{int})\text{Math.round}(y_{vp})$$

Pixels (x, y) in a framebuffer's viewport have integer coordinates that satisfy

$$0 \leq x \leq w - 1 \quad \text{and} \quad 0 \leq y \leq h - 1$$

with the pixel $(0, 0)$ being in the upper left hand corner.