1. Here are the three **trig substitutions**.

   If you have $\sqrt{a^2 - x^2}$ let $x = a\,\sin\theta$ so $\sqrt{a^2 - x^2}$ becomes $a\,\cos\theta$.

   If you have $\sqrt{a^2 + x^2}$ let $x = a\,\tan\theta$ so $\sqrt{a^2 + x^2}$ becomes $a\,\sec\theta$.

   If you have $\sqrt{x^2 - a^2}$ let $x = a\,\sec\theta$ so $\sqrt{x^2 - a^2}$ becomes $a\,\tan\theta$.

2. The trig substitutions are based on the Pythagorean trig identity

$$\cos^2\theta + \sin^2\theta = 1.$$

   From this identity we get the following three identities, which give us the three trig substitutions.

$$1 - \sin^2\theta = \cos^2\theta$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\sec^2\theta - 1 = \tan^2\theta$$

3. After you do a trig substitution, you get an integral that has only trig functions. When integrating trig functions, the following identities are often useful.

   The half-angle identities.

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

   The double-angle identities.

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

4. **Reduction Formulas**

$$\int u\sin u\,du = \sin u - u\,\cos u + C$$

$$\int u\cos u\,du = \cos u + u\,\sin u + C$$

$$\int u^n\sin u\,du = -u^n\cos u + n\int u^{n-1}\cos u\,du$$

$$\int u^n\cos u\,du = u^n\sin u - n\int u^{n-1}\sin u\,du$$