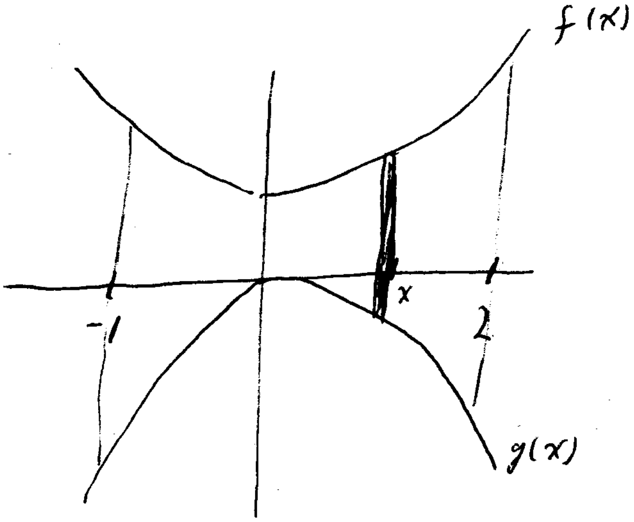
MA 16400 - Spring, 2014 Quiz 2 Name Solutions

1. Draw a sketch of the region bounded by the graphs of the two functions

$$f(x) = x^2 + 1$$

$$g(x) = \frac{-x^2}{2}$$

and the lines $x = -1$ and $x = 2$. Then set up the integral for the area of that region and find its value. Show your work.



$$\int_{-1}^{2} [(x^2+1) - (-\frac{x^2}{2})] \, dx$$

$$= \int_{-1}^{2} [x^2 + 1 + \frac{x^2}{2}] \, dx$$

$$= \int_{-1}^{2} \frac{3x^2}{2} + 1 \, dx$$

$$= [\frac{3}{2}(\frac{x^3}{3}) + x] \Big|_{-1}^{2}$$

$$= [\frac{1}{2}x^3 + x] \Big|_{-1}^{2}$$

$$= (\frac{8}{2} + 2) - (\frac{-1}{2} - 1)$$

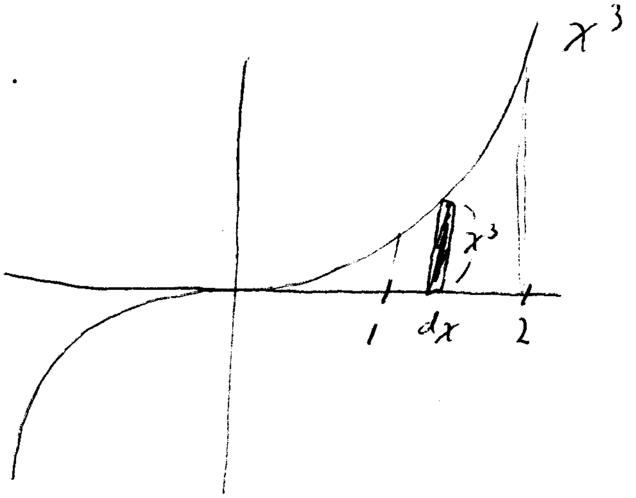
$$= 6 - (\frac{-3}{2})$$

$$= \frac{15}{2}$$

2. Draw a sketch of the region bounded by the x-axis, the lines $x = 1$ and $x = 2$, and the graph of the function

$$f(x) = x^3.$$

Show a typical vertical slice of the region and label it appropriately (what's its width, its height?). Set up the integral for the volume of revolution generated when that region is revolved around the x-axis and find its value. Show your work.



width = dx
height = x^3

$$\int_1^2 \pi\,(\text{radius})^2\,dx$$

$$= \int_1^2 \pi\,(x^3)^2\,dx$$

$$= \int_1^2 \pi\,x^6\,dx$$

$$= \pi\,\frac{x^7}{7}\Big|_1^2$$

$$= \frac{\pi}{7}\left(2^7 - 2^1\right)$$

$$= \frac{\pi}{7}\left(128 - 1\right) = \frac{127\pi}{7}$$