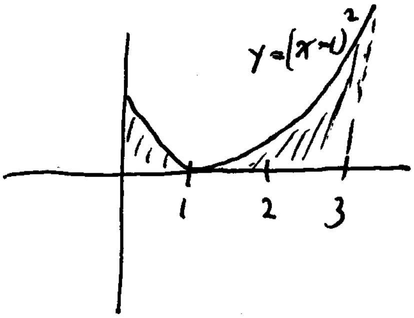
MA 16400 - Spring, 2014 Quiz 4 Name Solutions

1. Let R be the homogeneous lamina (with density 1) bounded by the graph of $y = (x-1)^2$, the x-axis, the y-axis, and the vertical line $x = 3$. Compute the value of $\bar{x}$ for this lamina.



$$\text{total moment} = \int_0^3 x(x-1)^2\,dx$$

$$= \int_0^3 x(x^2 - 2x + 1)\,dx$$

$$= \int_0^3 x^3 - 2x^2 + x\,dx$$

$$= \frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2}\Big|_0^3$$

$$= \frac{81}{4} - 18 + \frac{9}{2} = \frac{81 - 72 + 18}{4} = \frac{27}{4}$$

$$\text{mass} = \int_0^3 (x-1)^2\,dx = \int_{-1}^2 u^2\,du$$

$u = x-1$

$du = dx$

$$= \frac{u^3}{3}\Big|_{-1}^2 = \frac{8}{3} + \frac{1}{3} = \frac{9}{3} = 3$$

$$\bar{x} = \frac{27/4}{3} = \frac{9}{4}$$

2. Use the integration by parts technique,

$$\int u\,dv = uv - \int v\,du$$

to find the following antiderivative. Show your work.

$$\int (2x+3)\,e^x\,dx$$

$u = 2x+3$ $\quad$ $dv = e^x\,dx$

$du = 2\,dx$ $\quad$ $v = e^x$

$$= (2x+3)e^x - \int e^x\,2\,dx$$

$$= (2x+3)e^x - 2e^x + C$$

$$= 2xe^x + 3e^x - 2e^x + C$$

$$= 2xe^x + e^x + C$$

$$= (2x+1)e^x + C$$