1. For a function of two variables $f(x, y)$, a **vertical slice** is the intersection of the graph of $z = f(x, y)$ with a plane perpendicular to the xy-plane. Know how to use the idea of a vertical slice to describe what we mean by the "partial derivatives" of $f(x, y)$ at a given point (x_0, y_0).

2. For a function of two variables $f(x, y)$ and a point (x_0, y_0), the two vectors

$$(1,\ 0,\ \frac{\partial f}{\partial x}(x_0, y_0)) \quad \text{and} \quad (0,\ 1,\ \frac{\partial f}{\partial y}(x_0, y_0))$$

are both tangent vectors to the graph of $z = f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$.

3. The cross product of the two vectors from the previous item,

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & f_x(x_0, y_0) \\ 0 & 1 & f_y(x_0, y_0) \end{vmatrix} = (-\frac{\partial f}{\partial x}(x_0, y_0),\ -\frac{\partial f}{\partial y}(x_0, y_0),\ 1)$$

is perpendicular to the graph of $z = f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$, so it is a normal vector for the plane tangent to the graph of $z = f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$.

4. Let $\mathbf{n} = (-f_x(x_0, y_0),\ -f_y(x_0, y_0),\ 1)$, let $\mathbf{x_0} = (x_0, y_0, z_0)$ denote a given point on the graph of $z = f(x, y)$, and let $\mathbf{x} = (x, y, z)$ denote an unknown point on the plane tangent to the graph of $z = f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$. Then the vector equation $\mathbf{n} \cdot (\mathbf{x} - \mathbf{x_0}) = 0$ for this tangent plane simplifies to

$$z = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

5. The tangent plane formula is itself a function of x and y, $T_{(x_0,y_0)}(x, y)$, that we call the **tangent plane approximation function** for $f(x, y)$ at the point (x_0, y_0),

$$T_{(x_0,y_0)}(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

For arbitrary points (x, y) "near" the given point (x_0, y_0)

$$f(x, y) \approx T_{(x_0,y_0)}(x, y).$$

The tangent plane function approximates the original function.

6. Here is a picture illustrating the tangent plane approximation.

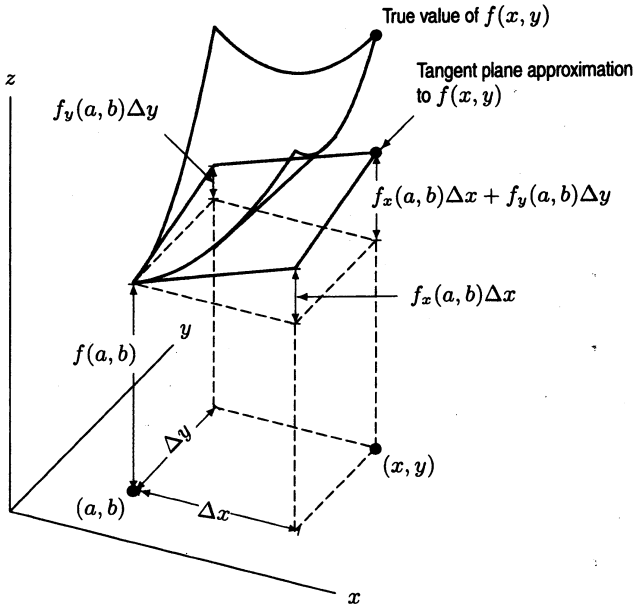

Figure 14.22: Local linearization: Approximating $f(x, y)$ by the z-value from the tangent plane

7. For a function of two variables $f(x, y)$, the **gradient vector** is defined to be the two-dimensional vector

$$\nabla f(x, y) = (\, f_x(x, y),\ f_y(x, y)\,) = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j}$$

8. Know the derivative rules for the gradient as given in Theorem B on page 642.

9. Let $\mathbf{x} = (x, y)$ denote an unknown point and let $\mathbf{x_0} = (x_0, y_0)$ denote a given point in the xy-plane. Then we can use the gradient vector to rewrite the formula for the plane tangent to the graph of $f(x, y) = f(\mathbf{x})$ at the point $(x_0, y_0, f(x_0, y_0)) = (\mathbf{x_0}, f(\mathbf{x_0}))$,

$$T(\mathbf{x}) = f(\mathbf{x_0}) + \nabla f(\mathbf{x_0}) \cdot (\mathbf{x} - \mathbf{x_0}).$$

Notice how much this looks like the formula for the tangent *line* at the point $(x_0, f(x_0))$ for a function of *one* variable $f(x)$,

$$T(x) = f(x_0) + f'(x_0)(x - x_0).$$