

1. Know the formula for the distance between two points,

$$|P_0P_1| = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$$

and know the equation for a sphere with radius r and center (x_0, y_0, z_0) ,

$$(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2 = r^2.$$

2. Know the algebra rules for vectors (Theorem A page 565, Theorem A page 577, Theorem C page 579).

3. Know how to compute the unit vector in the direction of \mathbf{v} (that is, $\frac{\mathbf{v}}{\|\mathbf{v}\|}$).

4. Know the algebraic and geometric definitions of the dot product,

$$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3 = \|\mathbf{u}\| \|\mathbf{v}\| \cos \theta$$

where θ is the angle between \mathbf{u} and \mathbf{v} .

5. Know that

a) $\mathbf{u} \cdot \mathbf{v} = 0$ means that \mathbf{u} and \mathbf{v} are perpendicular,

b) $\mathbf{u} \cdot \mathbf{u} = \|\mathbf{u}\|^2$.

6. Know the algebraic definition of the cross product

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}.$$

Know the geometric meaning of the cross product:

a) $\mathbf{u} \times \mathbf{v}$ is perpendicular to both \mathbf{u} and \mathbf{v} ,

b) $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\| \|\mathbf{v}\| \sin \theta =$ the area of the parallelogram determined by \mathbf{u} and \mathbf{v} .

7. Know that

a) $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ means that \mathbf{u} and \mathbf{v} are parallel,

b) $\mathbf{u} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{u})$, that is, the cross product is *anticommutative*,

c) the cross product is **not** associative. That is,

$$(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} \text{ is not equal to } \mathbf{u} \times (\mathbf{v} \times \mathbf{w}).$$

8. $\mathbf{i} \times \mathbf{j} = \mathbf{k}$ and $\mathbf{j} \times \mathbf{k} = \mathbf{i}$ and $\mathbf{k} \times \mathbf{i} = \mathbf{j}$.

9. The *projection* of \mathbf{v} onto \mathbf{u} is given by

$$\mathbf{proj}_{\mathbf{u}} \mathbf{v} = \left(\frac{\mathbf{v} \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}} \right) \mathbf{u} = \left(\frac{\mathbf{v} \cdot \mathbf{u}}{\|\mathbf{u}\|^2} \right) \mathbf{u} = \left(\frac{\mathbf{v} \cdot \mathbf{u}}{\|\mathbf{u}\|} \right) \frac{\mathbf{u}}{\|\mathbf{u}\|} = \left(\mathbf{v} \cdot \frac{\mathbf{u}}{\|\mathbf{u}\|} \right) \frac{\mathbf{u}}{\|\mathbf{u}\|}$$

and the *component* of \mathbf{v} in the direction of \mathbf{u} is given by

$$\text{comp}_{\mathbf{u}} \mathbf{v} = \frac{\mathbf{v} \cdot \mathbf{u}}{\|\mathbf{u}\|} = \|\mathbf{v}\| \cos \theta.$$

Notice that the projection is a vector and the component is a scalar.

10. Know the “scalar triple product”,

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = \text{the volume of the parallelepiped determined by } \mathbf{u}, \mathbf{v}, \text{ and } \mathbf{w}.$$

11. Know the equation for a plane in three-dimensional space,

$$\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}_0) = 0$$

where $\mathbf{p}_0 = (x_0, y_0, z_0)$ is a given point on the plane, $\mathbf{n} = (A, B, C)$ is a vector perpendicular (normal) to the plane, and $\mathbf{x} = (x, y, z)$ is a point that you are testing whether or not it is in the plane (makes the equation true). Another way to write this is

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0.$$

12. Know the parametric vector equation for a line in three-dimensional space,

$$\mathbf{r}(t) = \mathbf{r}_0 + t \mathbf{v}.$$

where $\mathbf{r}_0 = (x_0, y_0, z_0)$ is a given point on the line and $\mathbf{v} = (v_1, v_2, v_3)$ is a vector that points in the same direction as the line. Another way to write this is

$$x(t) = x_0 + tv_1 \quad y(t) = y_0 + tv_2 \quad z(t) = z_0 + tv_3.$$

13. Know that a *vector valued function* of a single variable

$$\mathbf{r}(t) = f(t) \mathbf{i} + g(t) \mathbf{j} + h(t) \mathbf{k}$$

represents a particle moving on a curve in space. The derivative

$$\mathbf{r}'(t) = f'(t) \mathbf{i} + g'(t) \mathbf{j} + h'(t) \mathbf{k}$$

is the velocity vector (which is a tangent vector to the curve of motion). The length of the velocity vector is the (instantaneous) speed of motion

$$\text{speed} = \|\mathbf{r}'(t)\| = \sqrt{\mathbf{r}'(t) \cdot \mathbf{r}'(t)} = \sqrt{f'(t)^2 + g'(t)^2 + h'(t)^2}$$

14. Know how to find the vector equation for the line tangent to a vector valued function $\mathbf{r}(t)$ at some given time t_0 .
15. The distance traveled, which is also called *arc length*, is given by the definite integral of the speed,

$$L = \int_{t_0}^{t_1} \sqrt{f'(t)^2 + g'(t)^2 + h'(t)^2} dt = \int_{t_0}^{t_1} (\text{instantaneous speed}) \times (\text{small interval of time})$$

16. Know the derivative rules for vector valued functions (Theorem B page 583), in particular, the *three* different product rules (why three?).