

## 1. Projection Transformation (Camera Space to Viewplane)

Let  $(x_c, y_c, z_c)$  be a point in camera coordinates and let  $(x_p, y_p, z_p)$  be its perspective projection onto the viewplane. Then

$$x_p = -x_c/z_c, \quad y_p = -y_c/z_c, \quad z_p = -1.$$

## 2. Clipping Equations (linear interpolation)

Let  $v_0$  and  $v_1$  be the endpoints of a line segment in the camera's viewplane  $z = -1$ . If the line segment crosses the boundary line  $x = 1$ , or  $x = -1$ , or  $y = 1$ , or  $y = -1$  (respectively), then we solve for  $t_c$  in one component of the equation

$$(1 - t) v_0 + t v_1 = (1, y_c)$$

$$(1 - t) v_0 + t v_1 = (-1, y_c)$$

$$(1 - t) v_0 + t v_1 = (x_c, 1)$$

$$(1 - t) v_0 + t v_1 = (x_c, -1)$$

(respectively) to find *when* the line segment crosses the boundary, and then use that  $t_c$  in the other component to find the point *where* the line segment should be clipped. If  $v_0$  and  $v_1$  have colors  $c_0$  and  $c_1$ , then the color of the new clipped point is given by

$$c = (1 - t_c) c_0 + t_c c_1$$

## 3. Viewplane to Pixel-plane Transformation

Let  $(x_p, y_p, -1)$  be a point in the camera's viewplane and let  $(x_{vp}, y_{vp})$  be its result of its transformation to the renderer's pixel-plane. Then

$$x_{vp} = 0.5 + (w/2)(x_p + 1),$$

$$y_{vp} = 0.5 + (h/2)(y_p + 1).$$

Points  $(x_p, y_p)$  from the camera's view rectangle have transformed pixel-plane coordinates that are contained in the *logical viewport* and satisfy

$$0.5 \leq x_{vp} < w + 0.5 \quad \text{and} \quad 0.5 \leq y_{vp} < h + 0.5$$

where  $w$  and  $h$  are the width and height of the framebuffer's viewport.

## 4. Pixel-plane to Pixel Transformation

Let  $(x_{vp}, y_{vp})$  be a point in the logical viewport of the renderer's pixel-plane and let  $(x, y)$  be the equivalent pixel in the framebuffer's viewport. Then

$$x = (\text{int})\text{Math.round}(x_{vp}) - 1$$

$$y = h - (\text{int})\text{Math.round}(y_{vp})$$

Pixels  $(x, y)$  in a framebuffer's viewport have integer coordinates that satisfy

$$0 \leq x \leq w - 1 \quad \text{and} \quad 0 \leq y \leq h - 1$$

with the pixel  $(0, 0)$  being in the upper left hand corner.