1. (10 points) Evaluate the following antiderivative. Show all your work.

$$\int \frac{x^3}{5x^4+3}\,dx$$

2. (10 points) Compute the following definite integral. Show all your work.

$$\int_1^4 \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx$$

3. Let R be the region bounded by the graphs of $f(x) = x(4 - x)$ and $g(x) = 2(4 - x)$.

(a) (12 points) Draw a sketch of the region R. Find the area of R. Show your work.

(b) (12 points) Write down, but do not evaluate, an integral for the volume of the solid generated when the region R is revolved about the horizontal line $y = -1$. Briefly explain how each part of your integral contributes to defining the total volume.

(c) (12 points) Write down, but do not evaluate, an integral for the volume of the solid generated when the region R is revolved about the vertical line $x = 2$. Briefly explain how each part of your integral contributes to defining the total volume.

4. (10 points) The natural length of a certain spring is 12 inches, and a force of 6 pounds is required to keep it stretched to a length of 14 inches. Find the work done in stretching the spring from a length of 14 inches to a length of 20 inches. Show your work.

5. (12 points) Write down, but do not evaluate, the integral for the surface area of the solid generated by revolving the graph of $y = e^{-x^2}$ around the x-axis between $x = -1$ and $x = 3$. Briefly explain how each part of your integral contributes to defining the total area.

6. (12 points) An upright cylindrical tank is 10 feet in diameter and 10 feet high. If water (with density 64.2 pounds per cubic foot) in the tank is 6 feet deep, how much work is done in pumping all the water over the top edge of the tank? Draw a sketch for this problem showing a small element of work. Set up the integral and briefly explain how each part of your integral contributes to computing the total work. Evaluate your integral, showing all your steps.

7. (10 points) Use the arc length formula

$$\int_a^b \sqrt{1 + f'(x)^2}\, dx$$

to find the length of the graph of the following function between $x = 0$ and $x = 1$.

$$f(x) = 1 + 2\, x^{3/2}$$