

1. Find each of the following antiderivatives. Show your work.

$$(a) \int x(3x^2 - 5)^{2/3} dx \quad \text{set } u = 3x^2 - 5$$

$$du = 6x dx$$

$$\frac{1}{6} du = x dx$$

$$= \int u^{2/3} \cdot \frac{1}{6} du$$

$$= \frac{1}{6} \int u^{2/3} du$$

$$= \frac{1}{6} \cdot \frac{1}{\frac{2}{3}+1} u^{\frac{2}{3}+1} + C$$

$$= \frac{1}{6} \cdot \frac{3}{5} u^{5/3} + C$$

$$= \frac{1}{10} (3x^2 - 5)^{5/3} + C$$

$$(b) \int (x\sqrt{x} + \frac{2}{x^{2/3}} + \frac{1}{4x}) dx = \int (x^{3/2} + 2 \cdot x^{-2/3} + \frac{1}{4} \cdot \frac{1}{x}) dx$$

$$= \int x^{3/2} dx + 2 \int x^{-2/3} dx + \frac{1}{4} \int \frac{1}{x} dx$$

$$= \frac{1}{1+3/2} x^{3/2+1} + 2 \cdot \frac{1}{-\frac{2}{3}+1} x^{-\frac{2}{3}+1} + \frac{1}{4} \ln|x| + C$$

$$= \frac{2}{5} x^{5/2} + 6x^{1/3} + \frac{1}{4} \ln|x| + C$$

2. Compute each of the following definite integrals. Show your work.

$$(a) \int_0^{\pi/4} \sin^4 x \cos x \, dx =$$

$$\text{set } u = \sin x$$

$$du = \cos x \, dx$$

change limits:

$$x=0 \rightarrow u = \sin 0 = 0$$

$$x = \frac{\pi}{4} \rightarrow u = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\int_0^{\frac{\sqrt{2}}{2}} u^4 \, du =$$

$$\frac{1}{5} u^5 \Big|_0^{\frac{\sqrt{2}}{2}} = \frac{1}{5} \left[\left(\frac{\sqrt{2}}{2} \right)^5 - 0 \right]$$

$$= \frac{1}{5} \cdot \frac{4\sqrt{2}}{2^5} = \frac{\sqrt{2}}{40}$$

$$(b) \int_1^2 e^{4x+2} \, dx$$

$$\text{set } u = 4x+2$$

$$du = 4 \, dx$$

$$\frac{1}{4} du = dx$$

change limits

$$x=0 \rightarrow u=6$$

$$x=2 \rightarrow u=10$$

$$= \int_6^{10} e^u \cdot \frac{1}{4} \, du$$

$$= \frac{1}{4} \left[e^u \Big|_6^{10} \right]$$

$$= \frac{1}{4} \left[e^{10} - e^6 \right]$$