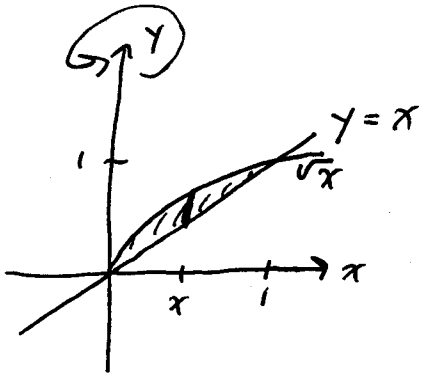
MA 16400 - Spring, 2014 Quiz 3 Name Solutions

1. Let R be the region bounded by the graph of $y = \sqrt{x}$ and the line $y = x$.

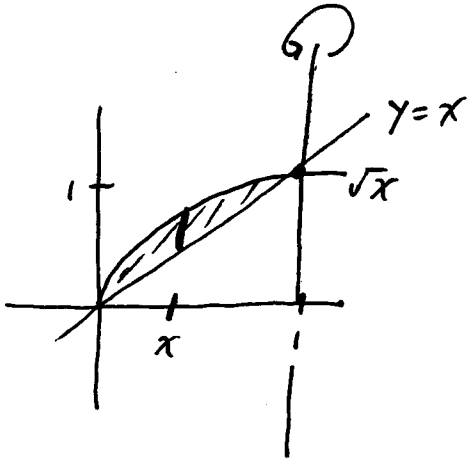
   (a) Using the shell method, set up the integral for the volume of the solid obtained by revolving the region R about the y-axis. Briefly explain how each part of your integral contributes to defining the total volume (for example, which parts represent circumference, area, thickness, etc.). Do not evaluate the integral.



$$\int_0^1 2\pi \,(\text{radius})(\text{height})(\text{thickness})$$

$$= \int_0^1 2\pi\, x\,(\sqrt{x} - x)\,dx$$

   (b) Using the shell method, set up the integral for the volume of the solid obtained by revolving the region R about the vertical line $x = 1$. Briefly explain how each part of your integral contributes to defining the total volume (for example, which parts represent circumference, area, thickness, etc.). Do not evaluate the integral.



$$\int_0^1 2\pi \,(\text{radius})(\text{height})(\text{thickness})$$

$$\int_0^1 2\pi\,(1-x)\,(\sqrt{x} - x)\,dx$$

(1 − x): radius of shell

(√x − x): height of shell

dx: thickness of shell

2π (radius of shell): circumference of shell

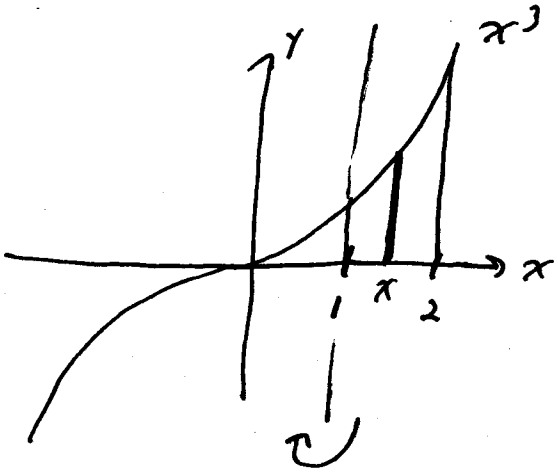
circumference of shell × height of shell: area of shell

area of shell × thickness of shell: volume of shell

2. Draw a sketch of the region bounded by the x-axis, the lines $x = 1$ and $x = 2$, and the graph of the function

$$f(x) = x^3.$$

Set up the integral (using the shell method) for the volume of the solid obtained by revolving that region about the vertical line $x = 1$ and find the value of the integral. Show your work.



$$\int_1^2 2\pi (x-1)\, x^3\, dx$$

2π (radius) $(x-1)$ (height) x^3 (thickness) dx

$$= 2\pi \int_1^2 x^4 - x^3\, dx$$

$$= 2\pi \left[\frac{x^5}{5} - \frac{x^4}{4} \right]_1^2$$

$$= 2\pi \left[\left(\frac{2^5}{5} - \frac{2^4}{4} \right) - \left(\frac{1}{5} - \frac{1}{4} \right) \right]$$

$$= 2\pi \left[\left(\frac{32}{5} - 4 \right) - \left(\frac{-1}{20} \right) \right]$$

$$= 2\pi \left[\frac{12}{5} + \frac{1}{20} \right] = 2\pi \frac{49}{20} = \boxed{\frac{49\pi}{10}}$$