MA 16400 - Spring, 2014 Quiz 5 Name Solutions

1. Find the following antiderivative. Show all your work.

$$\int (\sin^3 x)\sqrt{\cos x}\, dx$$

sin is to an odd power so

let $u = \cos x$

$du = -\sin x\, dx$

$-du = \sin x\, dx$

$$\int (\sin x)^3 (\cos x)^{\frac{1}{2}}\, dx = + \int (1-\cos^2 x)(\cos x)^{\frac{1}{2}} \sin x\, dx$$

$$= -\int (1-u^2)\, u^{1/2}\, du$$

$$= -\int u^{1/2} - u^{5/2}\, du$$

$$= \frac{-2}{3} u^{3/2} + \frac{2}{7} u^{7/2} + C$$

$$= \frac{-2}{3} \cos^{3/2} x + \frac{2}{7} \cos^{7/2} x + C$$

$$= \frac{-2}{3} (\cos x)^{3/2} + \frac{2}{7} (\cos x)^{7/2} + C$$

2. For the following integral, determine what would be the appropriate trig substitution. Then do the trig substitution to get an integral of trig functions. **Do not** do the final integration. (Hint: Complete the square.)

$$\int \frac{dx}{\sqrt{x^2+4x+5}}$$

Complete the square

$x^2+4x+4-4+5$

$(x^2+4x+4)+1$

$(x+2)^2+1$

$\boxed{u=\tan\theta}$

$du=\sec^2\theta\, d\theta$

$\int \frac{dx}{\sqrt{(x+2)^2+1}}$

$u=x+2$

$du=dx$

$$\int \frac{du}{\sqrt{u^2+1}} = \int \frac{\sec^2\theta\, d\theta}{\sec\theta} = \boxed{\int \sec\theta\, d\theta}$$

3. To find the antiderivative of the following function,

$$\int \frac{dx}{x^2\sqrt{4-x^2}}$$

we should use the trig substitution

$$x = 2\sin\theta \qquad \frac{x}{2}=\sin\theta \qquad \theta = \arcsin\left(\frac{x}{2}\right)$$

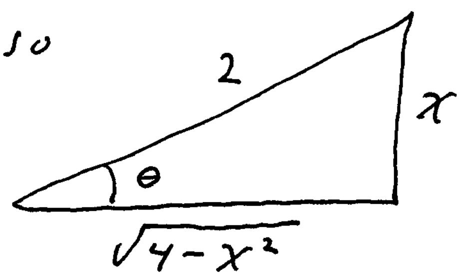
After doing the trig substitution and then evaluating the antiderivative in terms of the variable θ, we get the following.

$$\int \frac{dx}{x^2\sqrt{4-x^2}} = -\frac{1}{4}\cot\theta + C = \frac{-1}{4}\,\frac{\sqrt{4-x^2}}{x} + C$$

Finish this problem by doing the last step of converting the antiderivative back to the variable x. Show your work.

$\theta = \arcsin\left(\frac{x}{2}\right)$

so



$\cot\theta = \frac{\sqrt{4-x^2}}{x}$

$$\boxed{\frac{-\sqrt{4-x^2}}{4x} + C}$$