MA 16400 - Spring, 2014 Quiz 6 Name Solutions

1. Write the form of the partial fraction decomposition for the following rational function. *Do not* compute the unknown coefficients. (Hint: Factor.)

$$\frac{1}{x^2(x^2-4)(x^2+4)} = \frac{1}{x^2(x-2)(x+2)(x^2+4)}$$

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} + \frac{D}{x+2} + \frac{Ex+F}{x^2+4}$$

2. Use partial fraction decomposition to find the antiderivative of the following rational function. Show your work

$$\int \frac{1}{x^2+3x+2}\,dx = \int \frac{1}{(x+1)(x+2)}\,dx$$

$$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$1 = A(x+2) + B(x+1)$$

Let $x = -2$

$$1 = B(-1) \Rightarrow \boxed{B = -1}$$

Let $x = -1$

$$1 = A(1) \Rightarrow \boxed{A = 1}$$

$$\int \frac{1}{x+1} - \frac{1}{x+2}\,dx = \boxed{\ln(x+1) - \ln(x+2) + C}$$

$$= \boxed{\ln\left(\frac{x+1}{x+2}\right) + C}$$

3. Use partial fraction decomposition to find the antiderivative of the following rational function. Show your work

$$\int \frac{1-x}{x^3+x}\,dx = \int \frac{1-x}{x(x^2+1)}\,dx$$

$$\frac{1-x}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$1-x = A(x^2+1) + (Bx+C)x$$

Let $x = 0$

$$1 = A(1) \Rightarrow \boxed{A = 1}$$

Let $x = 1$ and $x = -1$

$$0 = 2 + (B+C) \Rightarrow -2 = B + C$$

$$2 = 2 + (-B+C) \Rightarrow 0 = C - B \Rightarrow B = C$$

So $-2 = C + C$ or $-2 = 2C$ so $C = -1$ and $B = -1$

$$\int \frac{1}{x} + \frac{-x-1}{x^2+1}\,dx = \int \frac{1}{x} - \frac{x+1}{x^2+1}\,dx$$

$$= \ln(x) - \int \frac{x\,dx}{x^2+1} - \int \frac{1}{x^2+1}\,dx$$

$u = x^2+1$

$\frac{1}{2}du = x\,dx$

$$= \ln(x) - \frac{1}{2}\ln(x^2+1) - \arctan(x) + C$$