Efficient Directional Network Backbone Construction in Mobile Ad Hoc Networks

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Abstract—In this paper, we consider the issue of constructing an energy-efficient virtual network backbone in mobile ad hoc networks (MANETs) for broadcasting applications using directional antennas. In directional antenna models, the transmission/reception range is divided into several sectors, and one or more sectors can be switched on for transmission. Therefore, data forwarding can be restricted to certain directions (sectors), and both energy consumption and interference can be reduced. We develop the notation of our directional network backbone by using the directional antenna model and form the problem of the directional connected dominating set (DCDS), which is an extreme case of the directional network backbone using an unlimited number of directional antennas. The minimum DCDS problem is proven to be NP-complete. A localized heuristic algorithm for constructing a small DCDS and two extensions of the algorithm are proposed. Performance analysis includes an analytical study in terms of an approximation ratio and a simulation study on the proposed algorithms by using both a custom simulator and ns2.

Index Terms—Connected dominating set (CDS), directional antennas, local solution, mobile ad hoc networks (MANETs), virtual network backbone.

1 INTRODUCTION

Broadcasting is the most frequently used operation in mobile ad hoc networks (MANETs) for the dissemination of data and control messages in the preliminary stages of many applications. Usually, a wired network backbone is constructed for efficient broadcasting, where only selected nodes that form the backbone forward data, and the entire network receives it. The dominating set (DS) has been widely used to select an efficient virtual network backbone. A set is dominating if every node in the network is either in the set or a neighbor of a node in the set. When a DS is connected, it is called a connected DS (CDS). In a CDS, any two nodes in the DS can be connected through intermediate nodes from the DS. Using a CDS, a connected virtual backbone has been widely used for efficient broadcasting in MANETs. In [13], it is demonstrated that any broadcast scheme based on a backbone with a size proportional to the minimum CDS (MCDS) guarantees a throughput within a constant factor of the broadcast capacity. CDS has been used in many other applications, including sensor coverage [1] and efficient communication using network coding [15].

In a directed graph, the set in the virtual network backbone for broadcasting is called the connected dominating and absorbent set [31]. If two nodes are connected by a directed edge, the start node is a dominating neighbor of the end node, and the end node is an absorbent neighbor of the start node. In a connected dominating and absorbent set, nodes in the set are strongly connected, and each node that is not in the set has at least one dominating neighbor and one absorbent neighbor in the set. As shown in Fig. 1a, black nodes \( \{u, v, w\} \) form a connected dominating and absorbent set. The set \( \{v, w\} \) is also strongly connected, and all the other nodes \( u \) and \( x \) can be dominated by it. However, \( x \) can only reach \( u \), which is not in the set; thus, the broadcast cannot achieve full coverage when the source is \( x \). \( \{v, w\} \) is not a connected dominating and absorbent set.

Recently, the directional antenna model [21] has been developed and implemented in various applications. With the help of switched beam and steerable beam techniques, antenna systems of wireless nodes can perform directional transmission and/or reception. A common directional antenna model involves dividing the transmission range of a node into \( K \) identical sectors, and one or more sectors can be switched on to transmit/receive. Compared with omnidirectional antenna systems, the use of directional antenna systems helps improve channel capacity and conserve energy, since the signal strength toward the direction of the receiver can be increased. Due to the constraints of the signal coverage area, interference can also be reduced.

In this paper, we put forth the directional network backbone concept. When using a directional antenna model, each node divides its omnidirectional transmission range into \( K \) sectors. Parts of them can be selected to be switched on for transmission. We assume that all nodes use a directional antenna for transmission and use an omnidirectional antenna for reception. A directional virtual network backbone is defined as a set of selected nodes and their associated selected transmission sectors. Only the nodes in the backbone forward data to their selected transmission sectors. The entire network receives the data, assuming the absence of interference. Fig. 1 illustrates the concept. The black nodes in Fig. 1a are a connected dominating and absorbent set that forms the
of efficient virtual network backbone, we propose the notion shown in Fig. 1.
where sectors are not necessarily aligned, unlike the case where it is 12. In this paper, we consider a general model this example. This is less than the original one in Fig. 1a, the total number of the selected sectors is 3 among black nodes in this example. This is less than the original one in Fig. 1a, where it is 12. In this paper, we consider a general model where sectors are not necessarily aligned, unlike the case shown in Fig. 1.

Inspired by the method of using a CDS to construct an efficient virtual network backbone, we propose the notion of directional CDS (DCDS) by using the directional antenna model, which is a special case of the directional network backbone, where $K$ is infinite. In a directed graph, a DCDS is a set of selected nodes and their associated selected edges. Each selected node can reach all other nodes, including nonselected nodes, via edges in DCDS. In addition, each nonselected node has an absorbent neighbor in the DCDS. We can see that with only nodes in the DCDS forwarding, the entire network will receive the broadcast data. Fig. 1c shows the DCDS in dark nodes and solid edges. There are five forwarding edges. This definition also works for undirected graphs, since they are special cases of directed graphs. When, in practice, the number of directional antennas of each node is finite, we can first find the DCDS. Then, each selected node simply switches on for the corresponding sectors that contain selected edges. We also develop a sector optimization (SO) algorithm. A minimum DCDS problem is finding one with the fewest selected edges. This is proven to be NP-complete in our paper. In contrast to the connected dominating and absorbent set, here we try to reduce forwarding edges as opposed to forwarding nodes. This guarantees the smallest energy consumption in the application of broadcasting by using directional antennas. Note that the energy consumption in any direction is fixed. The minimum DCDS problem is not a trivial extension of the MCDS problem. This is because there may be more nodes in the minimum DCDS than in the MCDS of a graph.

This paper focuses on using the DCDS concept to construct an energy-efficient directional backbone. We will focus on the following issues:

1. The directional network backbone problem. We put forward the concept of a directional network backbone, which includes a set of selected nodes and their associated selected transmission sectors using directional antennas, in order to reduce energy consumption and interference in MANETs.

2. The DCDS problem. We develop the DCDS problem, which is an extreme case of the directional network backbone problem, and prove the NP-completeness of the minimum DCDS problem.

3. Heuristic localized solutions to the minimum DCDS problem. We propose an approach for selecting forwarding nodes and edges for the minimum DCDS problem.

4. Optimization of transmission sectors. We present an optimization algorithm for determining transmission sectors, depending on the designated edges from DCDS when $K$ is finite.

5. Extensions of the proposed approach. We extend the proposed approach for a more energy-efficient DCDS by using an iterative scheme and apply it to topology control.

6. Performance analysis. We conduct performance analysis through analytical and simulated studies on the proposed solutions.

The remainder of this paper is organized as follows: Section 2 introduces some related work in the field. Section 3 presents the directional network backbone concept and then gives a new geometric graph model from which DCDS is defined. Section 4 presents the local heuristic algorithm for DCDS in directed graphs. Optimization of final transmission directions from designated transmission edges when $K$ is finite is also provided. Section 5 provides two possible extensions of the proposed algorithm. A performance study through simulation is conducted in Section 6. This paper concludes in Section 7.

2 RELATED WORK

We first review some related work on CDS construction approaches in MANETs, followed by an overview of directional antenna techniques and their applications.

2.1 General CDS Construction

The MCDS problem is NP-complete. Global solutions such as MCDS [6] and the greedy algorithm in [9] are based on global-state information and are expensive. The tree-based CDS approach [28] requires networkwide coordination, which causes slow convergence in large-scale networks. The cluster-based approaches in [35] are sequential algorithms. The status (clusterhead/nonclusterhead) of each node depends
on the status of its neighbors, which, in turn, depends on the status of the neighbors’ neighbors, and so on.

In local approaches, the status of each node depends on its $h$-hop information only with a small $h$, and there is no propagation of status information. Local CDS formation algorithms include Wu and Li’s marking process (MP) and self-pruning rule, Rules 1 and 2 [34], several MF variations [4], CEDAR [26], multipoint relay (MPR) [20], and MPR extensions [17]. In [4], the self-pruning rule, namely, Rule $k$, is proposed. This rule is a general form of Rules 1 and 2. In Rule $k$, a node can be withdrawn from the CDS if all of its neighbors are interconnected via $k (k \geq 1)$ nodes with higher priorities. The probabilistic approximation ratio of Rule $k$ is $O(1)$. Wu and Dai further propose the coverage condition for self pruning in [32], which can be viewed as a generic framework for several other existing broadcasting algorithms.

Most local solutions rely on node priorities to avoid simultaneous withdrawals in mutual coverage cases. One drawback of these priority-based schemes is that they may select a large CDS based on a bad priority assignment. Several iterative approaches [16], [33] have been proposed to find a small DS or CDS in MANETs. In [33], Wu et al. proposed a general framework of the iterative local solution for CDS. Their approach uses an iterative application of a selected local solution. Each application of the local solution enhances the result obtained from the previous iteration, but each is based on a different node priority scheme.

### 2.2 Directional Antennas

With the help of switched beam and steerable beam techniques [21], antenna systems can now form directional transmission and/or reception. We simply call them directional antennas, which is one type of smart antenna [30]. The most popular directional antenna model is ideally sectorized, as in [11], where the effective transmission range of each node is equally divided into $K$ nonoverlapping sectors, and one or more such sectors can be switched on for transmission or reception. The sectors of each node can be aligned, which means that sector $i \ (i = 1, \ldots, K)$ of all nodes points in the same direction. Another directional antenna model is the adjustable cone [25], using the steerable beam system. We use the ideally sectorized model in this paper, and we assume directional transmission and omnidirectional reception.

It is shown that the capacity of MANETs is reduced as the number of nodes increases if the system uses omnidirectional antennas [10]. The channel capacity when using directional antennas can be improved, because the directional transmission increases the signal energy toward the direction of the receiver. Also, the nodes can communicate simultaneously without interference. In [14], it is shown that the directional antenna technology has many features that help improve the spatial reuse of wireless channels. Directional antennas also permit greater frequency reuse and topology control and increase connectivity [3], [36].

Some probabilistic approaches for broadcasting using directional antennas are proposed. In [2], a broadcast scheme is proposed, using directional antennas to reduce redundancy. In [11], schemes are developed to switch off transmission beams toward known forwarding nodes or designate only one neighbor as a forwarding node in each direction. In [24], the directional version of Tseng et al.’s probabilistic protocols [27] is proposed, in which a node does not transmit toward a direction if this direction is covered by other nodes with high probabilities. Several centralized algorithms were proposed in [29], where a tree is built to connect all receivers with a minimal number of forwarding nodes and beam widths. Only two localized deterministic schemes were proposed [24], [25].

In [5], Dai and Wu proposed a deterministic localized broadcast protocol using directional antennas, where directional self pruning (DSP) was developed to reduce transmission directions. However, DSP is used for efficient broadcasting, where the source is known. All of the above schemes assume an omnidirectional reception mode. In [22], a wide spectrum of directional antenna models were analyzed. RF design and implementation of each model was discussed, and the minimum-energy broadcast algorithms for directional antennas were proposed. This broadcast incremental power (BIP)-based minimum-energy broadcast also deals with a given source broadcast.

### 3 Directional Connected Dominating Set

In MANETs, constructing a network backbone by selecting some nodes to forward helps achieve an efficient broadcasting procedure. Using the directional antenna model, a directional backbone can be constructed for broadcasting to further conserve energy and reduce interference.

In the directional antenna model, there is an edge connecting node $x$ to node $y$ if and only if (iff) $y$ is within the transmission range of $x$ and $y$ is in the sector of $x$, which is switched on. We assume, when using the omnidirectional model, that the given directed graph is strongly connected. The given graph can be an undirected graph as well, since it is a special case of a directed graph with symmetric connectivity, i.e., an edge $(u \rightarrow v)$ exists iff $(v \rightarrow u)$.

Neighborhood information is collected via exchanging “Hello” messages among neighbors. Here, we use a simple scheme for collecting two-hop information without using any location information (GPS). In directional neighbor- stationary discovery, each “Hello” message is sent out in every direction at each node, with the node ID and direction ID piggybacked in the message with the help of switched beam techniques. Note that the direction IDs of each node are fixed. By collecting “Hello” messages from its neighbors, each node $v$ can assemble its one-hop information, including a list of its neighbors and the directions used by those neighbors to reach $v$. $v$ can switch on the antenna in each direction for reception in turn. Thus, $v$ also gets the direction to reach each neighbor, i.e., the sector of $v$ in which each neighbor resides. The one-hop information of each node is exchanged among neighbors in the next round of “Hello” messages, and by assembling the one-hop information of all nodes, node $v$ can construct its two-hop information. Note that after the first “Hello” exchange, $v$ gets its dominating neighbors, and after the second one, $v$ gets its absorbent neighbors if they are also its dominating neighbors. The nodes that are only absorbent neighbors of $v$ may be detected by $v$ through three or more hops of information exchange. Neighborhood information that is still undetected can be ignored.
In the above scheme, each “Hello” message is sent out \( K \) times in \( K \) directions at each node. In traditional neighbor discovery schemes using omnidirectional “Hello” messages, each message is sent only once. However, given the same neighborhood area, the bandwidth and energy consumption of each directional transmission is roughly \( 1/K \) that of an omnidirectional transmission. The total cost of the directional neighborhood discovery is similar to that of the traditional scheme. This scheme also works when there are obstacles, as the neighbor and direction information is retrieved from a real signal reception instead of being computed from an ideal antenna pattern.

### 3.1 Directional Network Backbone

A directional backbone is a subset of nodes and their selected sectors such that each node in the backbone can reach any node in the original network by forwarding along the selected sectors. In addition, each node that is not part of the backbone can select a sector to reach a backbone neighbor. Note that the selection of a directional backbone may destroy the symmetric connectivity (of a given undirected graph), since the selection of \( (u \rightarrow v) \) does not coincide with the selection of \( (v \rightarrow u) \). That is, an undirected graph can become a directed one after the selection.

As shown in the example in Fig. 1b, the directional backbone contains three dark nodes and their selected sectors. The nodes that are not in the directional backbone are not used for forwarding. They are involved in the transmission only if they are the source. Each of them can use omnidirectional antennas for broadcasting for simplicity, or they can detect the sector that can reach a forwarding node and turn on the corresponding sector for transmission. Note that the derived graph of the directional backbone is a connected dominating and absorbent set. Thus, at least one such sector for each nonforwarding node exists.

The minimum directional backbone is the one with the minimum number of selected sectors. When \( K = 1 \), it is the traditional minimum connected dominating and absorbent set problem, where each sector corresponds to a node. Here, we consider another extreme case, i.e., when \( K = \infty \), where each edge becomes a sector.

### 3.2 Directed Connected Dominating Set

A CDS is usually used to construct an efficient virtual network backbone in MANETs. Inspired by this, we define a DCDS using directional antenna models to approximate the directional network backbone. The main idea is that in the directional virtual network backbone concept, if the number of sectors is infinite, the selection of switched-on sectors equals the selection of forwarding edges. Each outgoing edge of a node has a corresponding directional antenna and can be viewed as a transmission sector. In a directed graph, a directed edge from node \( u \) to node \( v \) is denoted as \( (u \rightarrow v) \), \( u \) is \( v \)'s dominating neighbor, and \( v \) is \( u \)'s absorbent neighbor. This edge is \( u \)'s dominating edge and \( v \)'s absorbent edge.

**Definition 1 (DCDS).** In a strongly connected directed graph \( G = (V, E) \), consider a subset of nodes \( V' \subseteq V \) and three subsets of edges \( E^s \subseteq \{(u \rightarrow v) | u,v \in V' \} \), \( E^d \subseteq \{(u \rightarrow v) | u \in V', v \in V - V' \} \), and \( E^a \subseteq \{(u \rightarrow v) | u \in V - V', v \in V' \} \) such that the following hold:

1. \( (V', E^s) \) is a strongly connected graph.
2. For \( v \in V - V' \), there exists \( u \), with \( (u \rightarrow v) \in E^d \).
3. For \( u \in V - V' \), there exists \( v \), with \( (u \rightarrow v) \in E^a \).

\( (V', E^s) \) is called a directional connected dominating and absorbent set, where \( E^s = E^d \cup E^a \) are the selected dominating edges of \( V' \).

**Definition 2 (The Minimum DCDS).** The minimum DCDS of a given graph is the one that has the smallest number of selected edges \( |E^s| \).

**Theorem 1.** The minimum DCDS problem is NP-complete.

**Proof.** Given any strongly connected graph \( G = (V, E) \), we can construct a new graph \( \overline{G} \) by adding an “image” vertex \( v' \) for each vertex \( v \) in \( V \) and two edges \( (v \rightarrow v') \) and \( (v' \rightarrow v) \), as shown in Fig. 2b, where \( (v \rightarrow v') \in E^d \), and \( (v' \rightarrow v) \in E^a \).

According to the definition of DCDS, \( (V, E^s \cup E^d) \) is a DCDS for \( \overline{G} \). Next, we prove that it is also the minimum DCDS for \( \overline{G} \). First, the node set \( V \) in the minimum DCDS is necessary. This is because any \( v \) in \( V \) needs to be included in the minimum DCDS. Otherwise, the corresponding \( v' \) has no dominating edge. Second, the node set \( V \) in the minimum DCDS is sufficient. This is because including any \( v' \) to the DCDS leads to the increase in the number of edges in the DCDS. That is, \( (v' \rightarrow v) \) needs to be in the DCDS.

Then, we prove that once \( V \), being the minimum forwarding node set, is determined, finding the minimum edge subset \( E^d \cup E^a \) in \( \overline{G} \) such that each node in \( V \) can reach all nodes in \( \overline{G} \) is NP-complete. In order to find \( E^d \cup E^a \), we need to first find \( E^s \) and then simply add \( E^d \) into the edge set. Note that \( |E^s| \) is a constant number that equals \( |V| \). The problem of finding the smallest strongly connected subgraph in terms of the number of edges in a given strongly connected graph \( G \) can be reduced to the Hamiltonian cycle problem and is
NP-complete [8]. Therefore, finding \( E^d \) is NP-complete, and so is finding \( E^u \cup E^d \).

We prove that, given any strongly connected graph \( G \), we can construct a new strongly connected graph \( \bar{G} \) in which the problem of finding the minimum DCDS is NP-complete. Therefore, the problem of finding the minimum DCDS is NP-complete in general.

The minimum DCDS problem in a unit disk graph is conjectured to be NP-complete. This is because we can also reduce the minimum DCDS problem to the problem of a Hamiltonian cycle in grid graphs with holes, which is NP-complete [19]. The grid graph is also a unit disk graph, since vertices are in some chosen integer coordinate points, and two nodes are connected if they are within a distance of one hop from each other.

Note that from the above proof, it is easy to prove that finding not only \( (V, E^u \cup E^d) \) but also \( (V, E^u) \) and \( (V, E^u \cup E^d \cup E^e) \) is NP-complete. The former is already included in the proof, and the latter can be proven by a similar approach, since \( |E^u| = |E^d| \).

Using omnidirectional antennas, the traditional connected dominating and absorbent set in directed graphs only focuses on the number of forwarding nodes. However, in DCDS, with the help of directional antennas, the number of forwarding edges determines the consumed energy. Hence, we are trying to find the DCDS with the minimal amount of forwarding edges. It is obvious that when \( K \) is infinite, the minimum DCDS corresponds to the minimum directional backbone. When \( K \) is finite, we can use a two-phase approach to approximate the minimum directional backbone. The first phase involves finding the minimum DCDS. In the second phase, each forwarding node switches on certain sectors covering all of its selected edges. A simple way of doing this is to switch on any sector that contains at least one selected edge. If the sectors of the directional antennas of each node are not necessarily aligned, an optimized sector selection algorithm can be designed, which will be discussed in the next section. Therefore, as shown in Fig. 1, from the result in Fig. 1c, the directional backbone as in Fig. 1b can be achieved.

### 4 Localized Heuristic Solution

We propose a heuristic localized approach for finding the minimum DCDS in directed graphs. A localized approach relies only on local information, i.e., properties of nodes within its vicinity. In addition, unlike the traditional distributed approach, there is no sequential propagation of any partial computation result in the localized approach. The status of each node depends on its \( h \)-hop topology only for a small constant \( h \) and is usually determined after \( h \) rounds of “Hello” message exchange among neighbors. A typical \( h \) value is 2 or 3. We use node priority to break the tie and avoid simultaneous node withdrawal. A node priority is unique. Different node properties can be used as the node priority, such as energy level, node ID, and node degree. We assume that the priority of node \( u \) is \( p(u) \) based on alphabetical order, such as \( p(u) > p(v) > p(w) > p(x) \) in Fig. 1. No location information is needed.

#### 4.1 Node and Edge Coverage Conditions

In [32], the coverage condition for CDS construction for undirected graphs states that a node \( v \) is unmarked if, for any two neighbors \( u \) and \( w \) of \( v \), a replacement path exists, connecting \( u \) and \( w \) such that each intermediate node on the path has a higher priority than \( v \). The coverage condition generates a CDS, since, for each withdrawn node, a replacement path for each pair of its neighbors must exist in order to guarantee the connectivity. Nodes in the replacement path can also cover neighbors of the withdrawn node.

The edge coverage condition (ECC) algorithm for DCDS, as shown in Algorithm 1, modifies the coverage condition concept to directed graphs. The main idea is to first select the forwarding nodes by using the node coverage condition, and then, each marked node applies the ECC to select forwarding edges. Note that although the procedure contains two phases, each node only collects the neighborhood information (topology and node priority) once (in the beginning). That is, further information exchange about node status (marked or unmarked) is not necessary.

**Algorithm 1: the ECC algorithm.**

1. Each node determines its status (marked/unmarked) by using the node coverage condition.
2. Each marked node uses the ECC to determine the status of its dominating edges.

**Node coverage condition.** Node \( v \) is unmarked if, for any two dominating and absorbent neighbors \( u \) and \( w \), a directed replacement path exists, connecting \( u \) to \( w \) such that

1. **each intermediate node on the replacement path has a higher priority than \( v \) if there is at least one and**
2. **\( u \) has a higher priority than \( v \) if there is no intermediate node.**

This node coverage condition is different from the coverage condition in [32] in the sense that when there is no intermediate node on the replacement path, \( v \) can be unmarked only if \( p(u) > p(v) \). Obviously, the node coverage condition is stronger than the original coverage condition. Thus, the marked nodes that it generated form a connected dominating and absorbent set. We will show later why this extra condition is necessary. Fig. 3a shows two types of directed replacement paths from \( u \) to \( w \), using the node coverage condition. When there is at least one intermediate node \( t \), \( p(t) > p(v) \). Otherwise, when \( u \) is directly connected to \( w \), \( p(u) > p(v) \) is necessary. We use the same concept for unmarked edges. Then, we introduce the priority assignment method for edges.
coverage condition), and for neighbor pair of the graph, and the ECC can be applied on every edge. Therefore, there is a total order for all the edges in the graph, and the priority of the end node. Thus, the priority of this edge is 

\[ \pi \in [0, 1] \]

For each edge \((v \rightarrow w)\), the priority of this edge is 

\[ p(v \rightarrow w) = (p(v), p(w)) \]

Thus, the priority of an edge is a tuple based on the lexicographic order. The first element is the priority of the start node of this edge, and the second one is the priority of the end node. Therefore, there is a total order for all the edges in the graph, and the ECC can be applied on every edge.

**ECC.** Edge \((v \rightarrow w)\) is unmarked if a directed replacement path exists, connecting \(v\) to \(w\) via several intermediate edges with higher priorities than \((v \rightarrow w)\).

Fig. 3b shows the directed replacement path for edge \((v \rightarrow w)\). In this case, both the intermediate edges \(((v \rightarrow u)\) and \((u \rightarrow w)\)) have higher priorities than edge \((v \rightarrow w)\). We still use Fig. 1 to illustrate the ECC algorithm. The forwarding nodes are marked as in Fig. 1b. Node \(x\) is unmarked, since for neighbor pair \(v, u\), there is a replacement path \((v \rightarrow u \rightarrow w)\), with \(p(u) > p(x)\) (case 1 of the node coverage condition), and for neighbor pair \(w, u\), there is a replacement path \((w \rightarrow u \rightarrow v)\), with \(p(u) > p(x)\) (case 2 of the node coverage condition). The dominating edges of the marked nodes are shown as solid lines. Note that the dominating edges of unmarked nodes can be omitted. Then, each marked node applies the ECC to determine the status of each dominating edge. In Fig. 1c, marked edges are shown in solid lines. For example, the edge \((w \rightarrow v)\) with priority \((p(w), p(v))\) is unmarked because of the replacement path \((u \rightarrow v \rightarrow w)\) with higher edge priorities \((p(u), p(v))\), \((p(u), p(v))\), \((p(v), p(u))\). The edge \((w \rightarrow x)\) with priority \((p(w), p(x))\) is unmarked because of the replacement path \((w \rightarrow u \rightarrow v \rightarrow x)\) with higher edge priorities \((p(u), p(v))\), \((p(u), p(v))\), and \((p(v), p(u))\). Note that when these two edges are unmarked, only two hops of local information is necessary.

**Theorem 2.** Given a directed graph \(G = (V, E)\), \(V'\) and \(E'\) generated by ECC construct a DCDS.

**Proof.** If we can prove that for any two nodes \(s \in V'\) and \(d \in V\), there is a path with all intermediate nodes and edges only from \(V'\) and \(E'\), we prove that \((V', E')\) is a DCDS. In ECC, after step 1, \(V'\) is a dominating and absorbent set; thus, there are paths connecting \(s\) to \(d\), with intermediate nodes all marked, as shown in Fig. 4a. We use set \(S_P\) to denote these paths. Now, we prove by contradiction. Suppose that any path in \(S_P\) connecting \(s\) to \(d\) has at least one unmarked edge (with a cross (X) on it). For each path in \(S_P\), we construct a subpath that satisfies the following: 1) it is a component of unmarked edges and 2) it is the closest to node \(d\). We then construct a subgraph containing all these subpaths. This subgraph forms an “outer rim” of node \(d\), as the shaded area \(W\) in Fig. 4a. We assume that edge \((u \rightarrow w)\) is the edge with the highest priority in area \(W\). Since \((u \rightarrow w)\) is unmarked, there must exist some replacement paths connecting \(u\) to \(w\) via edges with higher priorities than \(p(u \rightarrow w)\). We use set \(R_P\) to denote these replacement paths.

There are two cases for the status of nodes on paths in \(R_P\).

- **Case 1.** There is at least one path in \(R_P\) with only marked nodes on it. Since \(R_P\) connects \(u\) to \(w\), there is at least one edge on \(R_P\) that is also in \(W\). Then, we assume that edge \((u' \rightarrow w')\) is the edge on this path and also in \(W\). Therefore, \(p(u' \rightarrow w') > p(u \rightarrow w)\). This contradicts the assumption that \((u \rightarrow w)\) is the highest priority edge in area \(W\).

- **Case 2.** There is at least one unmarked node on each path in \(R_P\). As shown in Fig. 4b, these unmarked nodes form a rim \(W'\). We then assume that node \(u'\) has the highest priority in area \(W'\). Since \(u'\) is unmarked, a replacement path \(P_u\) must exist for it based on the node coverage condition:

- There is at least one node on \(P_u\), i.e., node \(u'\), which is also in \(W'\) (otherwise, there is a path in \(R_P\) with only marked nodes). The priority of \(u'\) is higher than that of \(u\), which contradicts the assumption that \(u'\) is the highest one in \(W'\).

- There is no intermediate node on \(P_u\). The dominating neighbor of \(u'\) is connected to its absorbent neighbor on \(P_u\) (\(a \rightarrow b\)) exists. If \(a \neq u\) or \(b \neq u, w\), \(w'\) can be removed from \(P_u\). If \(a = u\) and \(b = w\), since \(u'\) is unmarked, \(p(a) = p(u) > p(a')\), which contradicts the assumption that \(p(u' \rightarrow w) > p(u \rightarrow w)\) (edge \((u' \rightarrow w)\) is on \(P_u\)).

All of the contradictions above show that a path exists connecting \(s\) to \(d\) with only marked nodes and edges. \(\square\)

From the above proof, we can see why the second condition of the node coverage condition is necessary. In Fig. 4b, when \(a = u\), \(b = w\), and \(p(u') > p(a) = p(u)\) (thus, edge \((u \rightarrow w)\) can be unmarked based on the ECC), if \(u'\) can be unmarked based on the node coverage condition without the condition that \(p(a) = p(u) > p(u')\) as in the case in (2), \(u'\) and edge \((u \rightarrow w)\) are unmarked simultaneously.

Fig. 5 shows a large-scale example in a 10 × 10 area. There are 30 nodes, and the transmission range is 3. The resulting DCDS is shown as dark nodes and dark arrows. In the resulting graph, there are 13 forwarding nodes and 47 forwarding edges. All the directed neighbors of node 21 are connected to one another. For example, node 21 has the highest priority (the larger the node ID, the higher the node priority) in its local area. Thus, using the node coverage condition, it is a forwarding node. The same can be said for node 23.

### 4.2 Sector Optimization (SO)

After the directional edges are determined for each forwarding node, the transmission directions can be calculated based on the given number of sectors \(K\). We also assume that the sectors of the directional antenna of each node are not necessarily aligned. We can develop an
optimization algorithm to let each node circumgyrate its antennas to minimize the number of its switched-on sectors. The SO algorithm is shown in Algorithm 2.

Algorithm 2: the SO algorithm.
Align the edge of one sector to each selected forwarding edge and determine the one with the smallest number of switched-on sectors.

In Fig. 6, \( K = 4 \), and the forwarding node has four forwarding edges. The antenna sectors are circumgyrated to align with each edge. In cases Figs. 6c and 6d, there are a smaller number of switched-on sectors. The time complexity is the number of forwarding edges \( |E'| \) (dominating edges of node \( v \) in \( E' \)).

4.3 Property of ECC

We have shown the correctness of the proposed algorithm. We prove its effectiveness in this section.

We can easily show that the node coverage condition produces a smaller CDS than a known condition called Rule \( k \) [4]. Our previous work has proven that the expected number of marked nodes in Rule \( k \) is bounded by \( O(1)|CDS_{opt}| \). This is also an upper bound for the total number of marked nodes in ECC algorithms. When an ideally sectorized antenna model with \( K \) sectors is used, the expected number of transmission directions is \( O(K)|CDS_{opt}| \). Note that the above argument is applicable before the ECC is applied.

Theorem 3. Given an ideally sectorized antenna model with \( K \) sectors, the expected performance of the ECC algorithm is \( O(K) \) times greater than in an optimal solution in random MANETs.

Proof. Both the coverage condition, directed or undirected, and the node coverage condition produce a smaller CDS than a known condition called Rule \( k \) [4], with the assumption that nodes are randomly distributed to generate the geometric graph. In Rule \( k \), a node \( v \) can be unmarked if all its neighbors are interconnected via \( k (k \geq 1) \) nodes with higher priorities than \( v \). Obviously, this condition is stronger than both the coverage condition and the node coverage condition. Our previous work has proven that the expected number of marked nodes in Rule \( k \) is bounded by \( O(1)|CDS_{opt}| \). This is also an upper bound for the total number of marked nodes in the proposed algorithms. When an ideally sectorized antenna model with \( K \) sectors is used, the expected number of transmission directions is

\[
E[TDN] = O(K)|CDS_{opt}|
\]

Then, we consider an optimal solution with the minimal number of transmission directions \( TDN_{opt} \). For convenience, we denote any node with at least one transmission direction as a marked node. Obviously, all marked nodes form a CDS, denoted as \( CDS_{TD} \), and

\[
|CDS_{opt}| \leq |CDS_{TD}| \leq TDN_{opt}
\]

Combining (1) and (2), we have

\[
E[TDN] = O(1)TDN_{opt}
\]

The probabilistic bound is based on \( K \). Usually, \( K \) is a constant value, \( O(K) = O(1) \). Therefore,

\[
E[TDN] = O(1)TDN_{opt}
\]

5 Extensions

In this section, two extensions are proposed to further improve the energy efficiency of the proposed algorithm.
we can use also associate the priority with the energy level of the node to shifting, shuffling, or random, as proposed in [33]. We can in Fig. 7b. Thus, the edge ECDS after ECC. We assume that using some priority designation that can also be applied in our local solution for the dynamic environment via a special priority. Wu et al. [33] also proposed a seamless iterative algorithm can go on to execute the following steps. The number of iterations does not need to be too large, as proven in [33]. Then, the algorithm can go on to execute the following steps. Algorithm 3 is the iterative version of ECC (ECC-I).

Algorithm 3: the $k$-round ECC-I algorithm.
1. Execute ECC.
2. Exit if the number of iterations reaches $k$; otherwise, each node selects a new priority and exchanges status (and priority if needed) with neighbors.
3. Apply ECC again on marked nodes/edges. Only marked nodes/edges can be used as coverage nodes/edges to unmark other marked nodes/edges. Go to step 2.

For the priority rotation scheme, we can choose from shifting, shuffling, or random, as proposed in [33]. We can also associate the priority with the energy level of the node to make nodes with higher energy marked easily. For example, we can use (energy level, random number) as the node priority. Wu et al. [33] also proposed a seamless iterative local solution for the dynamic environment via a special priority designation that can also be applied in our algorithms. As shown in the example, Fig. 7a is a resulting ECDS after ECC. We assume that using some priority rotation scheme, the node priorities change to what is shown in Fig. 7b. Thus, the edge $(x \rightarrow v)$ can be unmarked. Fig. 7b is the reduced DCDS after the second round of ECC-I.

5.1 Iterative DCDS
As mentioned above, one of the drawbacks of all priority-based schemes is that they may select a large CDS based on a bad priority assignment. In the previous algorithms, a fixed priority of each node is used.

To avoid simultaneous withdrawals, the problem of a large selected set due to a bad priority assignment exists. Inspired by the iterative local approach in [33], we extend the proposed algorithm to iterative versions to mitigate the side effect of priority assignment.

In the proposed algorithm, which depends on node priority, some priority rotation schemes can be applied to generate a new priority for each node, and then the algorithm can be performed again. The number of iterations does not need to be too large, as proven in [33]. Then, the algorithm can go on to execute the following steps. Algorithm 3 is the iterative version of ECC (ECC-I).

Algorithm 4: the ECC-TC algorithm.
1. Use ECC to mark forwarding nodes and their forwarding edges.
2. Use “SO” to determine switched-on sectors of each forwarding node.
3. In each switched-on sector, set the transmission range to reach the farthest neighbor connected by a forwarding edge in this sector.

Compared with the energy-efficient broadcast protocol proposed in [12], where the DS is constructed from the result of the LMST-based topology control according to the “optimal radius,” ECC-TC first constructs the DCDS. Then, it applies topology control, setting up not only transmission ranges but also transmission directions. In Fig. 6d, final transmission ranges should be set to reach nodes $x$ and $q$ in the switched-on sectors, respectively.

6 Simulation
We evaluate the proposed algorithm ECC and its extensions, namely, ECC-I and ECC-TC via two groups of simulations conducted on a custom simulator and also the network simulator ns2 [7]. In the first group, we focus on the performance analysis by comparing the DCDS generated by the proposed algorithms with the traditional CDS using omnidirectional models in terms of the number of forwarding nodes, forwarding edges, switched-on sectors, and total power consumption in ideal networks without packet loss. In the second group, we analyze the efficiency and reliability of these algorithms when there is collision and mobility. We use two approaches to generate CDS, Rule $k$, and a coverage condition (Generic).

6.1 Simulation Environment
To generate a random network, $n$ nodes are randomly placed in a restricted $100 \times 100$ area. Networks that cannot form a strongly connected graph are discarded. The tunable parameters in the simulation are given as follows:

1. the number of nodes $n$ (we vary the number of deployed nodes from 20 to 160 to check the scalability of the algorithms),
2. the transmission range $r$ (in order to generate directed graphs, each node randomly picks its transmission range from 20 to 40),
3. the number of sectors of the antenna pattern $K$ (we use 4 and 6 as the values of $K$),
4. the number of hops $h$ (in coverage condition, two-, three-, or four-hop local information is collected for our localized algorithms),
5. the maximal forward jitter delay $d$ (we vary it from 0.01 to 100 ms), and
6. the average moving speed $v$.

When there is mobility, the average moving speed is varied from 1 to 25 m/s. The first four parameters are for the ideal network simulation, and the last two are for the realistic network simulation.
The following metrics are compared:
1. the number of forwarding nodes,
2. the number of forwarding edges,
3. the total energy consumption when used as topology control,
4. the energy reduction ratio, and
5. the delivery ratio of broadcast message in the realistic simulation.

The power consumption in ECC-TC is calculated according to the algorithm, and we use the square of the transmission range as the power consumption in one sector.

### 6.2 Simulation Results from Custom Simulator

Fig. 8 shows the comparison of Rule $k$ [4] and Generic [32], which generate the CDS, and the ECC algorithm, which generates the DCDS. $h$ is 2 in the following simulations, unless specified. In Fig. 8a, the number of forwarding nodes of ECC is larger than Generic. Rule $k$ is less efficient than Generic, so it generates a larger CDS, especially when the network is very dense. Fig. 8b is the comparison of the number of forwarding edges. In Rule $k$ and Generic, all the dominating edges of forwarding nodes are their forwarding edges. ECC has a much smaller number of forwarding edges than CDS, especially when $n$ is large. Figs. 8c and 8d show the numbers of switched-on sectors in Rule $k$ and Generic (all sectors of each forwarding node are switched on due to the omnidirectional antenna) and in ECC when $K$ is 4 and 6 respectively.

Fig. 9 shows the results of the three algorithms when the original graph is undirected ($r = 40$). Fig. 9a shows the selected forwarding nodes. Figs. 9b and 9d are the switched-on sectors when $K$ is 4 and 6, respectively. Compared with Fig. 8, a larger transmission range and more links lead to a smaller forwarding node set for all four algorithms. However, Rule $k$ and Generic have larger forwarding edge sets, because each forwarding node tends to have more edges. ECC has a smaller forwarding edge set than those in Fig. 8. In Figs. 9c and 9d, Rule $k$ and Generic have smaller switched-on sectors than in Fig. 8 due to the reduced number of forwarding nodes. Since the relative performance of the four algorithms is the same as in directed graph, we set the original graph to be directed in the following without loss of generality.

Fig. 10 shows the performance ECC with different $h$ values. Figs. 10a and 10b show the numbers of forwarding nodes and edges in the DCDS, with two-, three-, and four-hop local information. We can see that with more local information, the smaller DCDS can be achieved in terms of both forwarding nodes and edges. However, when $h$ increases from 3 to 4, the performance improvement is not significant. Thus, a relatively small $h$ is appropriate for the localized ECC. In ECC, the increase in $h$ helps reduce both forwarding nodes and forwarding edges.

Fig. 11 shows the performance analysis of the two extensions. Rule $k$ and Generic are also extended to iterative versions (Rule $k$-I and Generic-I) and applied for topology control purposes (Rule $k$-TC and Generic-TC). In Rule $k$-TC and Generic-TC, a dominating and absorbent set is constructed using Rule $k$ or Generic. Then, each marked node sets its transmission range to reach its farthest neighbor, and each unmarked node sets its transmission range to...
reach its nearest marked neighbor. Figs. 11a and 11b show the comparison of the numbers of forwarding nodes and forwarding edges in ECC, ECC-I, Rule $k$-I, and Generic-I. We can see that by using the iterative approach, the forwarding nodes and edges of ECC can be further reduced. However, the reduction in forwarding edges is not as significant as a reduction in forwarding nodes would be. This is because fewer forwarding nodes may cause each forwarding node to select more forwarding edges in order to form a DCDS. The performance of Rule $k$-I and Generic-I is still better than that of ECC-I in terms of the number of forwarding nodes. However, ECC-I is the best in terms of the number of forwarding edges. These results are consistent with those in Figs. 8a and 8b. Figs. 8c and 8d show the results of the total power consumption and reduction ratio when ECC, Rule $k$, and Generic are used for topology control. We can see that ECC-TC reduces the power consumption of the original network the most, especially when $n$ is large. Fig. 8d is the reduction ratio.

### 6.3 Simulation Results from ns2

In the previous simulation, we focused on the performance analysis of DCDS in terms of the number of forwarding nodes, forwarding edges, switched-on sectors, and total power consumption in our custom ideal networks. Next, we will study the efficiency and reliability of the algorithm in a realistic environment simulated by ns2 when there is collision and mobility.

Fig. 12 shows the simulation results by using ns2.1b9. We use the directional antenna model and an enhanced
IEEE 802.11 MAC layer provided by the enhanced network simulator (TeNs) [23]. We simulate the proposed ECC algorithm together with the blind flooding (Flooding), Rule $k$, and Generic algorithms to study their efficiency and reliability as functions of the network size, collision, and mobility. In the following simulation, $K$ is 4, and $h$ is 3. The
nodes share a single 2-Mbyte channel, and the traffic load is 1-10 packets per second (pps), with a packet size of 64 bytes. The “Hello” message interval is 1 second. We use the random waypoint mobility model [18].

Fig. 12a shows the percentage of the total switched-on sectors with different network sizes \((v = 1, d = 100)\). We can see that Flooding switches on almost all sectors for transmission, except when \(n\) is quite small, and the percentage of nodes that do not receive the packet due to collision and do not forward is relatively significant. All the other three algorithms have smaller percentages, which decrease as the number of nodes increases. Among them, ECC has the smallest percentage of switched-on sectors, and Rule \(k\) has the largest. These results are consistent with those in the ideal network simulation.

Figs. 12b, 12c, and 12d show the reliability in terms of delivery ratio. Fig. 12b has a different network size \((v = 1, d = 100)\). Flooding achieves a delivery ratio of almost 100 percent, especially when the network size is larger than 40. The other three have lower delivery ratios, and among them, ECC has the best performance. Fig. 12c has different forward jitter delay \((n = 100, v = 1)\). In order to reduce the collision caused by the directional hidden terminal problem, we use a random forward jitter delay in the simulation, which is within range \([0, d_{\text{max}}]\). We can see that when the jitter delay is small, for instance, \(d_{\text{max}} = 0.01 - 0.1\) ms, the delivery ratios of all the algorithms are very low, including Flooding. When \(d_{\text{max}}\) increases to 1, the delivery ratios are around 95 percent. When \(d_{\text{max}} = 100\), the delivery ratios reach the previous results as in Fig. 12b. However, the increase in \(d_{\text{max}}\) leads to the increase in the average end-to-end delay too. Fig. 12d has a different average moving speed \((n = 100, d = 100)\). Flooding is not affected by the mobility significantly due to its large redundancy. The other three have lower delivery ratios when the speed at which the nodes move increases. Among the three, Rule \(k\) has the largest delivery ratio when \(v\) is relatively large. This is because Rule \(k\) has the largest redundancy. However, the difference is quite small.

These four approaches present different trade-offs between energy efficiency and robustness. Blind flooding tolerates the node mobility the most while being the least efficient one. By developing the ECC, we are trying to exploit the other extreme on the spectrum, i.e., superior efficiency, but only for low-mobility networks.

### 6.4 Summary
The simulation results in this section can be summarized as follows:

1. ECC generates a DCDS with fewer forwarding edges than the Rule \(k\) and the coverage condition algorithms.
2. Using directional antennas, the number of switched-on sectors in ECC is smaller than those using omnidirectional antennas.
3. More local information helps improve the performance of ECC, but a relatively small \(h\) is sufficient.
4. ECC-TC reduces the performance of ECC; ECC-TC reduces the total power consumption of the original network significantly.

5. In a realistic network with collision, ECC outperforms Rule \(k\) and the coverage condition in terms of both efficiency and reliability.

6. With movement in the network, Rule \(k\) and the coverage condition algorithms are slightly better than ECC.

### 7 Conclusions
In this paper, we put forward the concept of directional network backbone. Using directional antennas, constructing a directional network backbone in MANETs can further reduce the total energy consumption and interference in broadcasting applications. A two-phase approach for the directional backbone is developed. First, we construct a DCDS, which is an extreme case of directional backbone, and then, selected edges in the DCDS are combined to form switched-on sectors. A heuristic localized algorithm for constructing a small DCDS is proposed, which is a nontrivial extension of the existing approaches for the regular DS problem. The SO algorithm is developed for the second phase. Then, two extensions of the proposed algorithm are discussed: one is the iterative version, and the other is the application that incorporates topology control. Performance analysis are conducted, including a theoretical analysis in terms of the approximation ratio and simulations of the proposed algorithms. In the future, we will develop localized solutions for the directional network backbone problem directly instead of a two-phase approach. We will also analyze the energy consumption issue in this problem, considering a practical energy model.

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