Scan-Based Movement-Assisted Sensor Deployment Methods in Wireless Sensor Networks [*]

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Abstract

The efficiency of sensor networks depends on the coverage of the monitoring area. Although in general a sufficient number of sensors are used to ensure a certain degree of redundancy in coverage, a good sensor deployment is still necessary to balance the workload of sensors. In a sensor network with locomotion facilities, sensors can move around to self-deploy. The movement-assisted sensor deployment deals with moving sensors from an initial unbalanced state to a balanced state. Therefore, various optimization problems can be defined to minimize different parameters, including total moving distance, total number of moves, communication/computation cost, and convergence rate. In this paper, we first propose a Hungarian algorithm based optimal solution, which is centralized. Then a localized Scan-based Movement-Assisted sensoR deploymenT method (SMART) and its several variations are proposed that use scan and dimension exchange to achieve a balanced state. An extended SMART is developed to address a unique problem called *communication holes* in sensor networks. Extensive simulation has been done to verify the effectiveness of the proposed scheme.

Keywords: *Dimension exchange, Hungarian method, load balance, movement-assisted, scan, sensor deployment, wireless sensor networks.*

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1 Introduction

Wireless sensor networks (WSNs) [1, 2] combine processing, sensing, and communications to form a distributed system capable of self-organizing, self-regulating, and self-repairing. The application of WSNs ranges from environmental monitoring, to surveillance, to coordinated target detection. The efficiency of a sensor network depends on the coverage of the monitoring area. Although, in general, a sufficient number of sensors are used to ensure a certain degree of redundancy in coverage so that sensors can rotate between active and sleep modes, a good sensor deployment is still necessary to balance the workload of sensors. Mobile sensors [3] can be exploited to provide a re-distribution.

After an initial random deployment of sensors in the field, the *movement-assisted sensor deployment* [4] can be applied, which uses a potential-field-based approach to move existing sensors by treating sensors as virtual particles, subject to virtual forces. Basically, the movement-assisted sensor deployment deals with moving sensors from an initial unbalanced state to a balanced state. Therefore, various optimization problems can be defined to minimize different parameters, including total moving distance, total number of moves, communication/computation cost, and convergence rate.

More recently, some extended virtual force methods, such as in [5] and [6] which are based on disk packing theory [7] and the virtual force field concept from robotics [8], are proposed. These methods simulate the attractive and repulsive force between particles. Sensors in a relatively dense region will explode slowly according to each other's repulsive force and head toward a sparse region. In this way, the whole monitoring area can achieve an even distribution of sensors. However, these methods may have a long deployment time since sensors move independently and they may even fail if all the sensors can achieve force balance but not load balance.

We assume that sensors are deployed randomly into the square monitoring area without considering of any physical obstacles. Then if we partition the monitoring area into many small regions, and use the number of sensors in a region as its load, the sensor deployment problem can be viewed as a load balance problem in traditional parallel processing, where each region corresponds to a processor and the number of sensors in a region corresponds to the load. The sensor deployment resembles the traditional load balance issue in parallel processing with several key differences:

- *Different objectives*. In traditional load balancing, total moving distance rather than the number of moves is important, whereas in sensor networks, the number of moves is also important because of relatively heavy energy consumption to start or stop a move.

- *Different technical issues*. One unique issue in sensor networks is the communication hole (or simply hole) problem where some regions of the network have no deployed sensors. Since there is no centralized control, the network can be partitioned. Therefore, the network needs to be connected first before load balancing.

In this paper, we first provide an optimal solution in 2-D meshes. This solution is based on the classic Hungarian method, but requires global information without considering sensor network connectivity. We then propose a method using a 2-dimensional (2-D) scan called Scan-based Movement-Assisted sensoR deploymenT method (SMART). A typical scan operation [9] involves a binary operator $\oplus$ and an ordered set $[w_0, w_1, ..., w_{n-1}]$ where each w_i represents the number of sensors in a region, and returns the ordered set $[w_0, (w_0 \oplus w_1), ..., (w_0 \oplus w_1 \oplus, ..., \oplus w_n)]$. In this paper, we consider only integer addition and boolean AND operations for scan. Using integer addition, the scan operation will return partial and total sum of the number of sensors. Since each region position and n are known, average load information can be easily calculated and distributed as can be the overload/underload situation of each ordered subset corresponding to a prefix of the ordered set.

In SMART, a given rectangular sensor field is first partitioned into a 2-D mesh through clustering. Each cluster corresponds to a square region and has a clusterhead which is in charge of bookkeeping and communication with adjacent clusterheads. A hybrid approach is used for load balancing, where the 2-D mesh is partitioned into 1-D arrays by row and by column. Two scans are used in sequence: one for all rows, followed by the other for all columns. Within each row and column, the scan operation is used to calculate the average load and then to determine the amount of overload and underload in clusters. Load is shifted from overloaded clusters to underloaded clusters in an optimal way to achieve a balanced state. By optimal, we mean the minimum number of moves and minimum total moving distance. By a balanced state, we refer to a state with the maximum cluster size (the number of sensors in a cluster) and the minimum cluster size being different by at most 1.

The *communication hole* problem in a 2-D mesh corresponds to a cluster with a cluster size of zero. Clearly, the scan approach cannot be used in a row or column with holes, since clusterheads separated by one or more holes cannot communicate with each other to perform a scan operation. In the extreme case, the 2-D mesh may be disconnected as shown in Figure 1, where the number in each circle corresponds to the cluster size, and sensors in each cluster can communicate with sensors in adjacent clusters as well as sensors in the same cluster. In Figure 1, the network is partitioned into two components. Our solution to the hole issue is based on planting a "seed" from a non-empty cluster

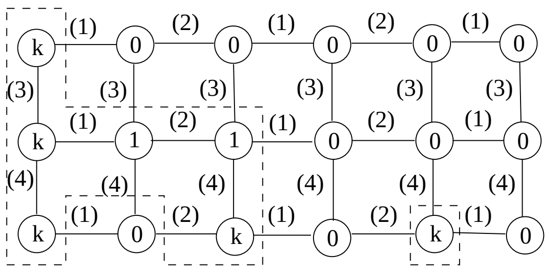


Figure 1: A sample clustered sensor network that corresponds to a 2-D mesh.

to an adjacent empty cluster. Various solutions are proposed in such a way that this seed-planting process (also called pre-processing) can be easily integrated with the normal 2-D scan process to achieve a good balance of various objectives. The network can use some newly developed location services [10, 11] to estimate the locations of sensors; thus no GPS service is required at each sensor and the corresponding overhead is avoided. For example, locations of sensors can be determined by using sensors themselves as landmarks [12].

The contributions of this paper are as follows: (1) We develop an optimal load balance solution based on the classic Hungarian method that achieves minimum total moving distance, and use it as a baseline to check the performance of other approaches. (2) We systematically discuss the similarity and difference between the traditional load balancing in parallel processing and movement-assisted sensor deployment in sensor networks. (3) We propose a new hybrid approach called SMART, together with several variations, that combines some desirable features of both local and global approaches while overcoming their drawbacks. (4) We identify a unique technical problem called communication hole and provide solutions to it. (5) We systematically study different trade-offs among various contradictory goals. (6) We conduct extensive simulations and compare results with several existing local movement-assisted sensor deployment methods.

2 Preliminaries and Related Works

2.1 Load balance in multiprocessor systems

Extensive work has been done in the parallel processing community on load balancing. In general, load balance algorithms can be classified as local (such as iterative nearest neighbor exchanging [13, 14]) and global (such as direct mapping [15, 16]). The global approach relies on global information which is usually not scalable. Local algorithms can be either deterministic or stochastic. Diffusion and dimension exchange are two widely used local deterministic methods. Both algorithms are iterative and are based on nearest neighbor exchange. Once all nodes complete one iteration, it is called a *sweep*. Although no information on load distribution is needed in local methods, iterative methods incur a significant number of rounds (moves in sensor networks).

In the diffusion method, the balancing procedure is divided into a sequence of synchronous steps. At each step, each node i interacts and exchanges load with all its neighbors, $adj(i)$. A diffusion parameter decides the portion of the excess load to be diffused between nodes i and each of its neighbors. Xu and Lau [17] proved that the optimal uniform diffusion parameter that leads to the fastest convergence for 2-D meshes is 1/4.

In the dimension exchange method, the edges of the graph are colored such that no two adjacent edges have the same color. A "dimension" is then defined as a collection of edges with the same color. In Figure 1, all edges are grouped into four dimensions. Edges with label (i) belong to dimension i ($i = 1, 2, 3, 4$). At each iteration, one particular color (dimension) is considered and every two adjacent nodes i and j connected by an edge with the selected color exchange their load according to an exchange rate. Again, Xu and Lau [17] showed the optimal uniform exchange rate for $2k_1 \times 2k_2$ 2-D meshes (where both row and column numbers are even).

2.2 Movement-assisted sensor deployment

The sensor placement issue has been researched recently [18], [19], [20]. Random placement of sensors may not satisfy the deployment requirement due to a hostile deployment environment. Therefore, the movement-assisted sensor deployment method is developed. Most existing movement-assisted protocols rely on the notion of virtual force to move existing sensors from an initial unbalanced state to a balanced state. These protocols are similar to nearest neighbor exchanging in load balancing. Sensors are involved in a sequence of computation (for a new position) and movement.

In [6], Zou and Chakrabarty proposed a centralized virtual force based mobile sensor deployment algorithm (VFA), which combines the idea of potential field and disk packing [7]. In VFA, there is a powerful clusterhead, which will communicate with all the other sensors, collect sensor position information, and calculate forces and desired position for each sensor. In VFA, the distance between two adjacent nodes when all nodes are evenly distributed is defined as a threshold to distinguish attractive or repulsive force between two nodes. The force between two nodes is zero if their distance is equal to the threshold, attractive if less than and repulsive if greater than. The total force on a node is the sum of all the forces given by other sensors together with obstacles and preferential coverage in the area. The clusterhead executes VFA and directs each sensor's movement. VFA has the drawbacks of centralized algorithms, single point failure, bottleneck of processing, and less scalability.

In [5], Wang, Cao, and La Porta developed a novel distributed self-deployment protocol for mobile sensors. They used Voronoi diagrams [21] to find coverage holes in the sensor network, and proposed three algorithms, VEC (Vector-based), VOR (Voronoi-based), and Minimax, to guide sensor movement toward the coverage hole. When applied to randomly deployed sensors, these algorithms can provide high coverage within a short time and limited moving distance. If the initial distribution of the sensors is extremely uneven, disconnection may occur, thus, the Voronoi polygon constructed may not be accurate enough, which results in more moves and larger moving distance. They adopted the optimization of random scattering of some sensors to cover holes. The termination condition of their algorithms is coverage instead of load balance. In [22], they further explored the motion capability of sensors for relocation to deal with sensor failure or respond to new events. The algorithm contains two phases. The first one is redundant sensor location, and the second is redundant sensor relocation. A grid-quorum solution was proposed to quickly locate the closest redundant sensors to the target area, where a sensor failure occurs. In their recent work [23], they designed a virtual movement scheme for the deployment protocol to reduce the moving distance of sensors. To our best knowledge, our work is the first to exploit scan-based movement assisted solution for sensor redistribution.

Some recent work focuses on sensors with limited mobility, which is motivated by the DARPA project called Intelligent Mobile Land Mine Units (IMLM) [24]. In IMLM, the mobility system is based on a hopping mechanism. Chellapan, Bai, Ma, and Xuan [25] studied a special hopping model in which each sensor can flip (or flop) only once to a new location. In addition, the flip distance is bounded. The deployment problem is then formulated as a minimum-cost maximum-flow problem.

3 An Optimal Solution

This section starts with an optimal solution for 2-D meshes based on the classic Hungarian method. Although due to its potential drawback of centralization, this optimal solution is not practical, especially when the WSNs are not connected, we can use it as a baseline to examine the performance of other proposed methods.

3.1 Hungarian method

Let us consider the *edge weighted matching problem* in a complete bipartite graph $K_{m,m}$ (m nodes on the left side and m on the right) with numbers associated with the edges called weights. The objective is to find a perfect matching (of m pairs), such that the sum of the weights of edges in the matching is maximum (or minimum). A matching is to find m edges to connect nodes on the left side to those on the right, and each node has only one edge.

A naive approach to solve the matching problem is to enumerate all m perfect matchings and find an optimal one among them. A better solution called the Hungarian method[1] exists. The following is the algebraic formulation for the matching problem. We let x_{ij}, ($i, j = 1, \ldots, m$), be a set of variables. m is the number of nodes in the node sets of the complete bipartite graph $B = (V, U, E)$, where V, U are two node sets, E is the edge set. $x_{ij} = 1$ means that the edge (v_i, u_j) is included in the matching, whereas $x_{ij} = 0$ means not. c_{ij} is the weight of edge (v_i, u_j). An optimal solution is to:

$$\begin{aligned} \text{Minimize} \quad & \Sigma_{ij} c_{ij} x_{ij} \\ \text{subject to} \quad & \textstyle\sum_{j=1} x_{ij} = 1 \quad i = 1, \ldots, m \\ & \textstyle\sum_{i=1} x_{ij} = 1 \quad j = 1, \ldots, m \end{aligned}$$

To use the Hungarian method to load balance in WSNs, we need to first convert the 2-D mesh to a complete bipartite graph using the follow procedure: (1) Calculate the global average $\bar{v}$ and determine “give”, “take”, and “neutral” state of each grid. (2) A node and edge weighted bipartite graph is constructed, where “give” and “take” grids appear at the left and right hand sides of the graph, respectively. The node weight corresponds to amount of overload and underload, and the edge weight represents the distance between the “give” and “take” grids in a matching pair. (3) An edge weighted perfect bipartite graph is derived by expanding each node with weight k to k “clone” nodes. The edge weight of clone nodes will inherit from the original nodes. It is obvious that the total sensor moving distance is minimized. The total number of moves is also minimized since each sensor, if necessary to move, only move once to its destination.

[1]In honor of the Hungarian mathematicians D. Kőnig and E. Egerváry who developed it.

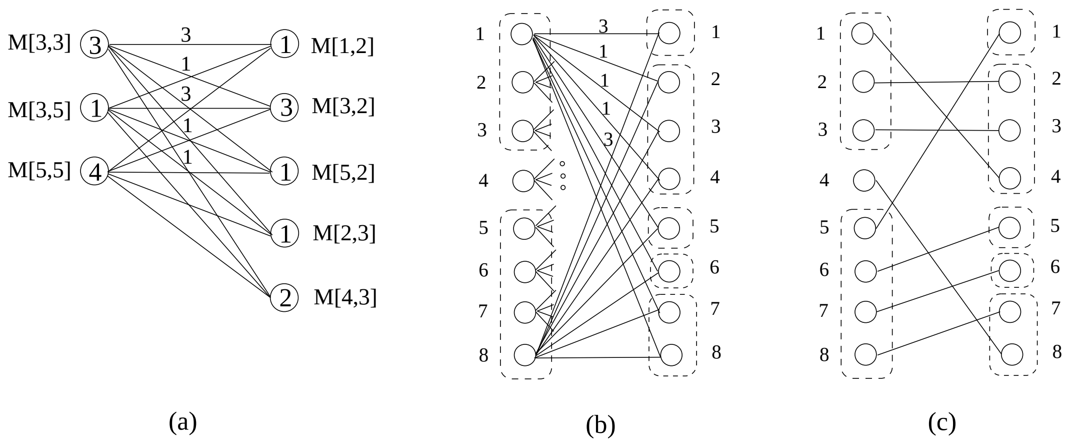


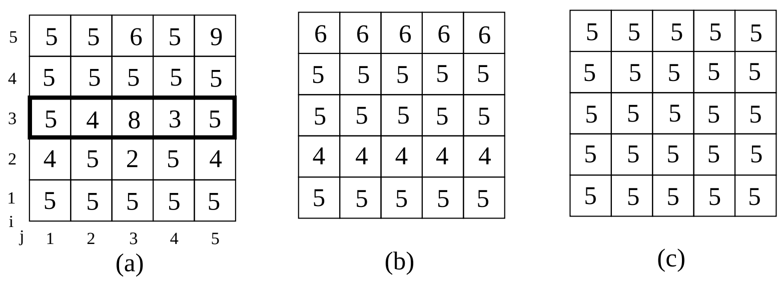
Figure 2: (a) The node and edge weighted bipartite graph of Figure 3 with "give" grids at the left-hand side and "take" grids at the right-hand side. (b) The edge weighted complete bipartite graph of (a) and (c) the optimal solution.

3.2 Examples and analysis

In Figure 3, the global average in case is 5. There are three overloaded nodes and five underloaded nodes. $M[3,3] = 3$ means overloaded by 3 units and $M[1,2] = 1$ is underloaded by 1 unit. The edge weight is the Manhattan distance between two end nodes $M[i, j]$ and $M[i^{'}, j^{'}]$. That is, $\Delta x + \Delta y = |i - i^{'}| + |j - j^{'}|$[2]. For example, the edge connecting $M[3,3]$ to $M[1,2]$ has a weight of 3. In Figure 2 (a), the node and edge weighted bipartite graph shows weights of all edges connecting $M[3,3]$ to underloaded nodes. In Figure 2 (b), the edge weighted complete bipartite graph of (a) is shown, where each node (overloaded or underloaded) with weight k has k "clone" nodes. For example, $M[3,3]$ has three clone nodes labeled from 1 to 3. The Hungarian method is then applied to (b) and the optimal result is shown in (c). The optimal result shows that $M[5,5]$ (now with four clone nodes) needs to move one sensor to each of $M[1,2]$, $M[5,2]$, $M[2,3]$, and $M[4,3]$.

There are several polynomial implementations for the Hungarian method. Our implementation is based on Munkres' method [26]. Another implementation [27] solves the problem in $O(m^3)$, exploiting the solution to the maximum flow problem. The cost of implementing the Hungarian method for load balance in WSNs is $O(m^3)$, where m is the amount of overloads (underloads) which

[2]The general distance between two points is defined as $((\Delta x)^k + (\Delta y)^k)^{1/k}$. When $k = 2$, it is Euclidian distance, and when $k = 1$, it is Manhattan distance.



(a)

i \ j	1	2	3	4	5
5	5	5	6	5	9
4	5	5	5	5	5
3	5	4	8	3	5
2	4	5	2	5	4
1	5	5	5	5	5

(b)

6	6	6	6	6
5	5	5	5	5
5	5	5	5	5
4	4	4	4	4
5	5	5	5	5

(c)

5	5	5	5	5
5	5	5	5	5
5	5	5	5	5
5	5	5	5	5
5	5	5	5	5

Figure 3: An ideal case for SMART.

is bounded by the number of sensors. Usually, the number of sensors is one or two magnitudes higher than the number of grids (n). A BS (base station) is needed to connected to the WSN, serving as the central controller for information collection and algorithm execution. Then BS informs all clusterheads about the sensor movement via direct or multi-hop communication.

4 SMART

4.1 Basic ideas

Unlike the optimal solution, SMART is a hybrid of the local and global approach. Its extension (discussed in then next section) can be used in disconnected WSNs. The sensor network is partitioned into an $n \times n$ 2-D mesh of clusters (the method can be easily extended to the general $n \times m$ 2-D mesh). Each cluster covers a small square area, and is controlled by a clusterhead. The role of each clusterhead can be rotated within the cluster. Each clusterhead, in charge of communication with adjacent clusters, knows the following information: (1) its cluster's position, i, in the 2-D mesh (via GPS) and (2) the number of sensors, w_i, in the cluster.

Two rounds of balancing are used with one for each dimension, first row, then column. As shown in Figure 3, after the first round, all rows are balanced in (b); after the second round, all columns are balanced, as is the whole area. Although balancing within a row or column can be done either locally such as iterative nearest neighbor interaction or globally such as direct mapping, SMART relies on an extended scan method.

4.2 Clustering

Since each sensor node knows its cluster id, i, sensors in the same cluster elect a unique clusterhead based on a pre-defined priority. Assume each cluster covers an $x \times x$ square. To ensure the square is covered whenever there is a sensor in the region, the sensing range r_1 should be set to $\sqrt{2}x$ (the diagonal length of the square). To support transmission from non-clusterhead to clusterhead, the intra-cluster transmission range should be set to at least $\sqrt{2}x$ (also denoted as r_1). To ensure the clusterhead can communicate with clusterheads in four adjacent clusters, the inter-cluster transmission ranges of each clusterhead should be at least the diagonal of the rectangle constructed from two adjacent squares. That is, $r_2 = \sqrt{5}x$. If a sensor does not support two transmission ranges, r_2 can be used for intra-cluster communication.

Generally, the role of clusterhead should rotate among all the nodes in the cluster to achieve balanced energy consumption and to prolong the life span of each individual node, such as in [28]. Non-clusterheads only need to report their own position and energy to clusterheads using transmission range r_1, while clusterheads will communicate with neighboring clusters, take over the information of sensors in its cluster, and direct the movement of sensors.

4.3 Scan

Consider the 1-D array of clusters where cluster id is labelled following the sequence in the linear line. Again, denote w_i as the number of sensors in cluster i. Let v_i be the prefix sum of the first i clusters, i.e., $v_i = \sum_{j=1}^{i} w_j$. $v_n = \sum_{j=1}^{n} w_j$ is the total sum. Clearly, $\overline{w} = v_n/n$ is the average number of sensors in a balanced state, and $\overline{v_i} = i\overline{w}$ is the prefix sum in the balanced state. Note that $\overline{w}$ is a real number which should be rounded to an integer $\lfloor \overline{w} \rfloor$ or $\lceil \overline{w} \rceil$. In a balanced state, $|w_i - w_j| \leq 1$ for any two clusters in the network.

The scan algorithm works from one end of the array to another (first scan) and then from the other end back to the initial end (second scan). The direction of the first sweep is called *positive* (with increasing order of cluster id) and that of the second sweep *negative* (with decreasing order of cluster id). The first sweep calculates the prefix sum v_i, where each clusterhead i determines its prefix sum v_i by adding $v_{i-1} + w_i$ and forwarding v_i to the next cluster. The clusterhead in the last cluster determines v_n and $\overline{w} = v_n/n$ (load in a balanced state) and initiates the second scan by sending out $\overline{w}$. During this scan, each clusterhead can determine $\overline{v_i} = i\overline{w}$ (load of prefix sum in a balanced state) based on $\overline{w}$

Table 1: The scan process on the third row of Figure 3.

i	1	2	3	4	5
w_i	5	4	8	3	5
v_i	5	9	17	20	25
$\overline{v_i}$	5	10	15	20	25

that is passed around and its own cluster position i.

Knowing the load in the balanced state, each cluster can easily determine its “give/take” state. Specifically, when $w_i - \overline{w} = 0$, cluster i is in the “neutral” state. When $w_i - \overline{w} > 0$, it is overloaded and in the “give” state; and when $w_i - \overline{w} < 0$, it is underloaded and in the “take” state. Each cluster in the give state also needs to determine the number of sensors (load) to be sent to each direction: $w_i^{\rightarrow}$ for load in the positive direction (or simply give-right) and ${}^{\leftarrow}w_i$ for load in the negative direction (give-left). Based on the scan procedure, we have

$$w_i^{\rightarrow} = \min\{w_i - \overline{w}, \max\{v_i - \overline{v_i}, 0\}\} \tag{1}$$

$$^{\leftarrow}w_i = (w_i - \overline{w}) - w_i^{\rightarrow} \tag{2}$$

The 2-D scan process involves a row scan followed by a column scan as shown in Figures 3 (b) and 3 (c), respectively. Table 1 shows details of the row scan on the third row where i is the column number. Only the cluster at columns 3 is in the “give” state, since its load is higher than $\overline{w} = 5$. For column 3, $w_3^{\rightarrow} = 2$ (the load will be assigned to column 4, the actual schedule will be discussed later) and ${}^{\leftarrow}w_3 = 1$ (it will be assigned to column 2). Similarly, a set of conditions can be given for the “take” state: $w_i^{\leftarrow}$ for take-right and ${}^{\rightarrow}w_i$ for take-left. It is clear that

$$^{\rightarrow}w_i = \min\{\overline{w} - w_i, \max\{v_{i-1} - \overline{v_{i-1}}, 0\}\} \tag{3}$$

$$w_i^{\leftarrow} = (\overline{w} - w_i) -^{\rightarrow} w_i \tag{4}$$

In the subsequent discussion, we use ${}^{\rightarrow}w_i$ for both the number of take-left units and the take-left state of cluster i. The same convention is used for the other three notations. The distinguishing feature of scan is its simplicity, where each clusterhead in i passes only one package in each sweep, prefix sum w_i in one sweep followed by global average $\overline{w}$ in the second sweep.

4.4 Properties of Scan

An optimal load balance scheduling based on scan should satisfy the above four conditions related to give-right, give-left, take-right, and take-left for each cluster. By optimal, we mean the minimum number of moves and minimum total moving distance. The following theorem shows that any violation of the conditions will result in the increase of overall moving distance and/or total number of moves to reach a load balanced state.

Theorem 1: *Any violation of the four conditions on give and take state of each cluster will result in the increase of overall moving distance and/or total number of moves to reach a load balanced state.*

Proof: We consider four types of violation: take state changed to give state, give state changed to take state, take-right (take-left) changed to take-left (take-right), and give-right (give-left) changed to give-left (give-right).

Suppose cluster i's state is changed from take to give and one unit is sent to cluster j. To ensure load balancing, that one unit at cluster i will be compensated by another unit from cluster k (i.e., k gives one unit back to i). A better scheme would be k giving one unit directly to j to save one move, and shorten the distance if j and k are at the same side of i in the 1-D array.

Suppose cluster i's state is changed from give to take and one unit is given from cluster j. To ensure load balancing, that one unit will be given away to cluster k. It would be better for j to give one unit directly to k to save one move, and shorten the distance if k and j are at the same side of i.

When cluster i's state mixes give-right with give-left, we assume that one unit is moved from $w_i^{\rightarrow}$ to ${}^{\leftarrow}w_i$ (similarly for ${}^{\leftarrow}w_i$ to $w_i^{\rightarrow}$). We show that this schedule will generate a longer moving distance. Suppose this unit is moved from i to $i^{'}$ ($1 \leq i^{'} < i$), based on the balanced state requirement, one unit in a cluster j in region $[1..i-1]$ needs to be moved out to cluster $j^{'}$ with $i < j^{'} \leq n$. We consider swapping these two units at i and j. To compare moving distance between these two cases (before and after the swap), we consider two situations shown in Figure 4 as follows

1. When $i^{'} \leq j < i$, we have $|i - i^{'}| + |j - j^{'}| > |j - i^{'}| + |i - j^{'}|$.
2. When $1 \leq j < i^{'}$, we have $|i - i^{'}| + |j - j^{'}| = |i - i^{'}| + |j - i^{'}| + |i^{'} - j| > |j - i| + |i - j|$.

In both cases, the moving distance before the swap $|i - i^{'}| + |j - j^{'}|$ is longer than that of after the swap.

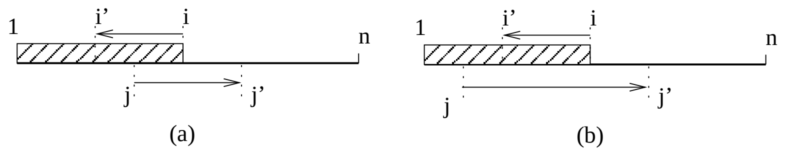


Figure 4: Two cases for mixing up give-right with give-left.

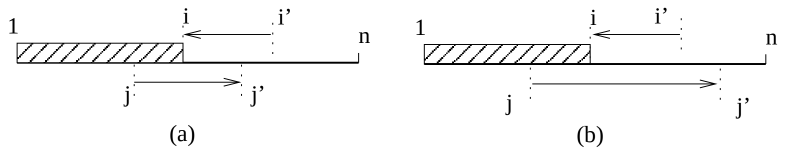


Figure 5: Two cases for mixing up take-right with take-left.

When cluster i's state mixes take-right with take-left, we again assume that one unit is moved from $^{\rightarrow}w_i$ to $w_i^{\leftarrow}$ (similarly for $w_i^{\leftarrow}$ to $^{\rightarrow}w_i$). Suppose this unit is moved from $i^{'}$ to i ($i < i^{'} \leq n$), based on the balanced state requirement, one unit in a cluster j in region $[1..i-1]$ needs to be moved out to cluster $j^{'}$ with $n \geq j^{'} > i$. We consider swapping these two units at i and j. To compare moving distance between these two cases (before and after the swap), we consider two situations shown in Figure 5 for $j^{'} \leq i^{'}$ and $j^{'} > i^{'}$. Following the similar argument as in the above case, the moving distance before the swap $|i - i^{'}| + |j - j^{'}|$ is longer than that of after the swap. □

The following theorem shows that when four conditions are met, overall moving distance is independent of the actual schedule.

Theorem 2: *When take-right (take-left) states get load from give-left (give-right) states, the overall moving distance is independent of the actual schedule.*

Proof: Let's consider schedules for all take-right states that get load from give-left states. The take-left states getting load from give-right states case can be argued in a similar way. Starting from cluster 1 and checking towards cluster n (i.e., along the positive direction), for each unit of underload in a take-right state i, assign one unit of load from the closest give-left state $i^{'}$ (i.e., a cluster in a give-left state with minimum id). Now we show that all other assignments can be converted to the above schedule without changing the total moving distance. Suppose in the above state, the unit to i comes from a non-closest give-left state $j^{'}$ and the unit from $i^{'}$ is assigned to a take-right state j where $i \leq j \leq i^{'}$. By swapping $i^{'}$ with $j^{'}$, total moving distance remains the same, and the one unit in i now

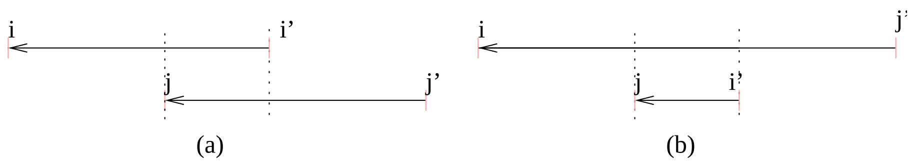


Figure 6: Swapping of i' and j' without changing the total moving distance: (a) before the swap, and (b) after the swap.

comes from i' (see Figure 6). This kind of swap can be done iteratively. □

4.5 An optimal scan in 1-D arrays and its extension in 2-D meshes

In this subsection, we propose a simple sender-initiated optimal load balance algorithm for 1-D arrays. The unique property is that the algorithm starts from each cluster in give state (give-left and give-right) in parallel without the need to be concerned with the detail of take state of other clusters. Suppose i is in a take state where $\bar{w} - w_i > 0$, then we do not distinguish take-right from take-left.

Sender-Initiated Optimal Load Balance in 1-D Arrays

1. For each cluster i in give state, the clusterhead sends $w_i^{\rightarrow}$ units to its right neighbor and sends ${}^{\leftarrow}w_i$ units to its left neighbor.

2. For each cluster i in take state, when the clusterhead senses several bypassing units, it intersects as many units as possible to fill in its "holes". Unintersected units move along the same direction.

Theorem 3: *The proposed greedy schedule ensures an optimal schedule in 1-D arrays.*

Proof: It suffices to show the case in Figure 5 is avoided. That is, the two conditions related to take state are satisfied. Based on the algorithm, when a unit is passed to i from right to left as shown in Figure 5, it implies that subarray $[i...n]$ is in overloaded state; similarly, when a unit is passed to j' from left to right, the subarray $[1...j']$ is in overloaded state. Since $i < j'$, the array $[1...n]$ as a whole is overloaded, which corresponds to a contradiction. □.

When the scan procedure is extended from 1-D arrays to 2-D meshes, the scan procedure is applied twice: once on all rows, followed by once on all columns. This 2-D scan process represents the core of SMART. However, this approach is no longer optimal in 2-D meshes. For example, consider a 2×2 mesh $M[1,1] = 3, M[1,2] = 1, M[2,1] = 3$, and $M[2,2] = 5$. A scan on rows will change load distribution of the mesh to $M[1,1] = 2, M[1,2] = 2, M[2,1] = 4$, and $M[2,2] = 4$, and a scan on columns will balance the mesh to $M[1,1] = 3, M[1,2] = 3, M[2,1] = 3$, and $M[2,2] = 3$. A total of 4 moves occur, however, the optimal solution requires only 2 moves from $M[2,2]$ to $M[1,2]$ directly.

Theorem 4: *The ratio between the 2-D scan and the optimal solution in terms of the number of moves is bounded by 2.*

Proof: During the 2-D scan, wasted moves occur during the first scan when a (globally) underloaded cluster i moves the load to another (globally) underloaded cluster j. Suppose L units of load are moved from i and j. L units of load for j are necessary, while L units for i are wasted units. A similar situation occurs when a (globally) overloaded cluster i moves load to another (globally) overloaded cluster j. In this case, L units for j are wasted, while L units for i are necessary. It is easy to follow that for each wasted move there is a matching necessary move, therefore, the ratio is bounded by 2. $\square$

4.6 Several variations of SMART

In SMART, an "aggressive" approach is used where a local "give" state in a row or column can be a global "take" state. To avoid this situation, a "conservative" approach can be used to decide local "give" and "take" state based on global average information.

Besides the prefix sum of the first i grids in a row (or column) in the positive direction, i.e., $v_i = \sum_{j=1}^{i} w_j$, another negative direction prefix sum is exploited, where $v'_i = \sum_{j=i}^{n} w_j$, and $v'_1 = \sum_{j=1}^{n} w_j$ is the total sum in the row (or column). The negative prefix sum is achieved in the negative sweep where the average is sending out. Now, $\overline{w_l} = v_n/n$ is the average number of sensors in a local balanced state with respect to the current row (or column). $v = \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij}$ is the global total sum. Then $\overline{w_g} = v/n^2$ is the average number of sensors in a global balanced state. We define a third kind of average as $\overline{w_m} = |\overline{w_g} - \overline{w_l}|/2$, the mean of global and local balanced state. This average is to achieve a compromise between conservative and aggressive approaches.

The variation differs from the original SMART in its definition of threshold $\overline{w}$ used to determine the "give/take" state. Still, when $w_i - \overline{w} = 0$, grid i is in the "neutral" state. When $w_i - \overline{w} > 0$, it is overloaded and in the "give" state; and when $w_i - \overline{w} < 0$, it is underloaded and in the "take" state. $\overline{w}$ can be one of three possible choices: $\overline{w_l}$, $\overline{w_g}$, and $\overline{w_m}$. Again, $\overline{v_i} = i\overline{w}$ is the the prefix sum in the balanced state under the given threshold $\overline{w}$, and $\overline{v_i}' = (n-i+1)\overline{w}$ is that of the negative direction. $\overline{w}$ should be rounded to an integer.

In the original SMART, the threshold is based on the local average, $\overline{w_l}$, when "give" and "take" states are balanced in a row (or column). With a changing threshold, such a balance is no longer held. That is, there could be more "give" than "take" grids and vice versa. Therefore, $w_i^{\rightarrow}$ for load in the positive direction (or simply give-right) and ${}^{\leftarrow}w_i$ for load in the negative direction (give-left) are changed as follows: a grid is in "give" state if its value is over the given threshold $\overline{w}$. The amount of excessive load to be transferred to its right (or left) depends on the amount of underload to its right (or left) provided that amount does not cause the underload of the current node. More formally, we have

$$w_i^{\rightarrow} = \min\{w_i - \overline{w}, \max\{\overline{v'_{i+1}} - v'_{i+1}, 0\}\} \quad (5)$$

$${}^{\leftarrow}w_i = \min\{(w_i - \overline{w}) - w_i^{\rightarrow}, \max\{(\overline{v_{i-1}} - v_{i-1}), 0\}\} \quad (6)$$

The threshold-based scan approach

1. If $\overline{w} \neq \overline{w_l}$, determine global balanced value $\overline{w_g}$.
2. Perform a row scan followed by a column scan using the selected $\overline{w}$.
3. If $\overline{w} \neq \overline{w_l}$, repeat step (2) using $\overline{w} = \overline{w_l}$.

$\overline{w_g}$ in step (1) can be calculated during step (2). Basically, $\overline{w_g}$ is determined after row and then column scans. However, in these scans there are no actual sensor movements. Movements occur once $\overline{w}$ is derived from $\overline{w_g}$. Step (3) is needed since the result of step (2) cannot guarantee a globally balanced state. When $\overline{w} = \overline{w_m}$, one variation of the algorithm is to repeat step (2) a constant (c) number of times before applying step (3). We use SMART(g), SMART(l), and SMART(m, c) to represent the threshold-based scan that uses global average, local average (the original SMART), and mean of global and local average, respectively. c in SMART(m, c) corresponds to the number of iterations of step (2).

If the total number of sensors is unknown, more information propagation is necessary. After the last cluster of each row gets the total number in its row, one more scan is generated in the last column to achieve the global average. Then a scan in the negative direction in the column is conducted to distribute the average to each row.

5 Extended SMART

5.1 Simple solutions

The 2-D scan discussed previously works only when there is no hole, otherwise, certain rows and columns may not be connected. In the worst case, the 2-D mesh may be disconnected. A pre-processing is needed to plant “seeds” to holes at each 1-D scan and these seeds will serve as clusterheads in these holes.

Planting seeds in holes in an asymptotically optimal way is a non-trivial task. Suppose we want to optimize total moving distance, the number of moves, and communication latency (where each sequential neighbor communication is considered one step). The total moving distance should be $O(n^2)$ (as in the case of the first row of Figure 1), the number of moves should be $O(n)$, and communication latency should be $O(n)$.

A conservative approach could be sending out one seed at a time to an adjacent empty cluster. This will work for the case of the third row of Figure 1 where k is a number larger than 5 and the direction is from left to right. However, this approach does not work well for the case of the first row, since the frontier node, the clusterhead of the first nonempty cluster in the expansion direction, needs to communicate with the left most node after each probing and expansion. The corresponding communication latency is $2\sum_{i=1}^{n-1} i = O(n^2)$. Note that if the moving distance is a dominating factor, rather than the communication latency, this is still an acceptable solution.

In an aggressive approach, each cluster that has a sufficient number of sensors (seeds) can send out multiple seeds to cover the rest. This approach certainly works for the case of the first row, but fails for the case of the third row. In this case, the total moving distance would be $(n-1)^2 + (n-3)^2 + ... + 3^2 + 1^2 = O(n^3)$ since clusters in give state can initiate the process simultaneously. Also the number of moves is $(n-1) + (n-3) + ... + 3 + 1 = O(n^2)$.

The simple recursive doubling does not work either for the case of the second row, where the span of each expansion is doubled in the subsequent step. This is because $\log n$ expansions will incur at least $n/2 \times \log n = O(n \log n)$ communication latency, assuming the initial span is 1.

5.2 Optimal seed planting in 1-D arrays with holes

We propose a solution for the hole issue that is asymptotically optimal for several parameters, including communication latency ($O(n)$), total moves ($O(n)$), and total moving distance ($O(n^2)$), assuming that each cluster knows only the state of its two neighbors through probing. It is also assumed that the sensor network is sufficiently dense such that global $\overline{w} \geq 2$ (i.e., on average, each cluster has 2 sensors). Later we will resort to a slightly stronger condition when the solution is extended from 1-D arrays to 2-D meshes.

First, we give some notations used in the solution. A *segment*, S_i, is a maximum sequence of non-empty clusters. W_i is the summation of load in S_i and C_i is the length of S_i. Now we introduce two important concepts related to S_i:

- *Expansion level*, L_i, of S_i: $2^{L_i} \leq C_i < 2^{L_i+1}$.
- *Energy level*, E_i, of S_i: $E_i = W_i - C_i$.

Expansion level L_i determines spans of successive expansions 2^{L_i}, 2^{L_i+1}, 2^{L_i+2}, ... , whereas energy level E_i indicates the number of denotable sensors in the segment. E_i should be large enough to cover holes in each expansion, i.e., $E_i \geq 2^{L_i+k}$ for the kth expansion, which is called the *expansion condition*. Any cluster that has more than one sensor is in a denotable state for providing seeds, even though the cluster may be in an underloaded state.

The solution is based on recursive doubling of the span for each successive expansion until there is no sufficient energy for expansion, but the actual size of expansion is governed by the current expansion level. For segment S_i with level L_i, the sequence of span is 2^{L_i}, 2^{L_i+1}, 2^{L_i+2}, ... For example, suppose the length C_i of S_i is 13, the first span is $2^3 = 8$, making the new segment with length 21; the next expansion with span $2^4 = 16$ will increase the length to 29, and so on.

Two approaches, reactive or proactive, can be used here. In the reactive approach, each cluster waits for an expansion signal from one of its predecessors or until a pre-defined time-out expires (the time-out value is given in Theorem 5 below). This approach trades potential long delay for small total moving distance and total moves. This approach operates in the synchronized environment, where the synchronization point can be set during the initial deployment phase. In the proactive approach, each segment acts independently for expansion. This approach has minimum communication latency but with occasional extra sensor movements for the lack of synchronization. The solution can be described by the following steps: (1) Following the positive direction, each segment performs expansion through recursive doubling, when either it is informed from a predecessor segment or a predefined timeout expires in the reactive approach, or without waiting for any signal or timeout for activation in the proactive approach, until it either reaches the last cluster of the 1-D array or fails the expansion condition. (2) Repeat step 1. for the negative direction except no timeout is needed at this step.

The efficiency of the method depends on the worst case timeout in the reactive approach and excessive movement in parallel seed-planting in the proactive approach. The next theorem shows that it is sufficient to set timeout to $5(i-1)$, where i is the id of the first cluster in the segment. The total moving distance in the proactive approach is still bounded within $O(n^2)$.

Theorem 5: *In each segment S in a scan, the total moving distance in constructing S is bounded by C^2 and the communication latency is bounded by $5C$.*

Proof: We prove by induction, when S_i expands to connect S_j to form a new S_k along the positive direction, we assume that $C_i^{'}$ is the span S_i used to connect S_j and $C_j^{'}$ is the span of the non-overlapping region in S_j as in Figure 7. Note that S_i may merge with another segment S_j to form a new segment, S_k, as the result of the expansion of S_i (as shown in Figure 7). S_k will calculate its W_k and L_k accordingly. The special case S_j does not exist and has the length 0. The following proof still applies.

Based on the induction, the latency in the formation of S_i is bounded by $5C_i$. In the current expansion, C_i is needed for the frontier node to inform all relevant clusters along the negative direction in S_i and it takes $C_i + C_i^{'}$ time to pass seeds to relevant positions. Finally, it takes $C_j^{'}$ steps to reach the frontier of S_k (i.e., the right most node in S_j). Using the fact that $C_i^{'} \leq C_i < 2C_i^{'}$ (expansion conditions), we have $5C_i + C_i + (C_i + C_i^{'}) + C_j^{'} < 5(C_i + C_i^{'} + C_j^{'}) = 5C_k$.

Similarly, we show total moving distance by induction. Based on the induction, the formation of S_i is bounded by C_i^2. In the current expansion, the total moving distance is bounded by $\sum_{l=0}^{C_i^{'}-1}(C_i + l) = C_iC_i^{'} + C_i^{'}(C_i^{'} - 1)/2$. In the proactive approach, the formation of S_j needs to be included which is bounded by $C_j^2 < (C_i^{'} + C_j^{'})^2$. Using the fact that $C_i^{'} \leq C_i < 2C_i^{'}$, we have $C_i^2 + C_iC_i^{'} + C_i^{'}(C_i^{'} - 1)/2 + (C_i^{'} + C_j^{'})^2 < (C_i + C_i^{'} + C_j^{'})^2 = C_k^2$ □

Since the method involves two sweeps, the overall moving distance is clearly bounded by $O(n^2)$

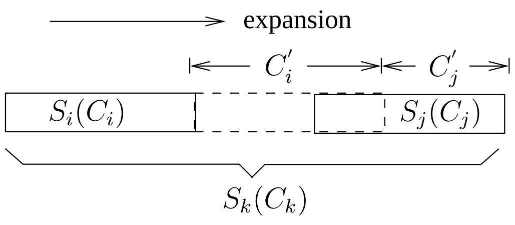


Figure 7: The merging of two segments.

and the overall communication latency is bounded by $O(n)$. Total moves are bounded by $O(n)$ in the reactive approach, and by $O(n \log n)$ in the proactive approach. In the latter case, clusters can plant seeds in parallel, but recursive doubling limits parallel merging to $\log n$ levels of the merging tree. Therefore, the proposed method in the proactive mode is optimal for the three parameters.

The following theorem shows that no timeout is needed in the second scan and proves the correctness of the 1-D scan approach. The postfix of the 1-D array is a subarray that contains the last cluster in the array.

Theorem 6: *Assume the average load is at least 2 for each cluster. After the first scan, at least one postfix of the 1-D array is a segment. In the second scan, no timeout is needed. All holes will be filled.*

Proof: It is assumed that average load for each cluster is at least 2. Suppose $S_1, S_2, ..., S_{k-1}, S_k$ is the sequence of segments after step 1 of pre-processing, where for each S_i (except S_k), $E_i < 2^{L_i}$, that is, $W_i < 2C_i$. If we let $\sum_{i=1}^{k-1} W_i = W_M$ and $\sum_{i=1}^{k-1} C_i = C_M$, we have $W_M < 2C_M$. Based on the assumption of at least average load of 2 for each cluster, we have $W_M + W_k \geq 2C_M + 2C_k > W_M + 2C_k$, therefore, $W_k > 2C_k$. S_k has sufficient energy for expansion. The only case for preventing such an expansion is that S_k includes the last cluster in the 1-D array. Therefore, S_k is a postfix of the 1-D array.

During step 2 of pre-processing, since S_k has sufficient energy, it will fill in the "gap" (a consecutive sequence of empty clusters) between S_k and S_{k-1} by planting seeds in holes between them. Following the same argument, the newly formed segment will have sufficient energy to fill the next gap. In this way, all gaps will be filled after the second scan. □

The result from Theorem 6 shows that the scan process can be combined with the pre-processing (planting the seeds). That is, the scan process can start at step 2 of the pre-processing.

5.3 Extended SMART

Now let us extend the approach from 1-D to 2-D. The first issue is to ensure that each 1-D row array in the 2-D mesh meets $\overline{w} \geq 2$. Instead of enforcing it (which is impossible), we propose a smoothing process on all columns before the pre-processing on rows. The smoothing process on columns includes pre-processing (i.e., plant seeds in holes) and scan (i.e., load balance). This column-wise smoothing process does not completely remove holes or balance load along columns unless the number of sensors in each column is at least $2n$ initially. However, when the sensor network is sufficiently dense, each row will have $\overline{w} \geq 2$ after the column-wise smoothing process. The following theorem shows the density requirement.

Theorem 7 *Suppose the average number of sensors in a cluster is at least 4. After column-wise smoothing, each row will have at least $2n$ sensors.*

Proof: We try to find the maximum number of sensors that can be deployed when at least one row still has less than $2n$ sensors after column-wise smoothing. If that number is less than $4n^2$, the theorem is proven.

Assume initially k columns have load of at least $2n$ and the remaining $n-k$ columns have load under $2n$. The former k columns will achieve load balancing after smoothing, while the latter $n-k$ columns will not. Without loss of generality, we assume row 1 (i.e., first nodes in all columns) has less than $2n$ sensors after smoothing. All the first nodes of those $n-k$ columns that have not achieved the balanced state are holes. The maximum total load of nodes other than the first nodes in these $n-k$ columns is bounded by $(n-k)(2n-1)$. The loads of first nodes of the other k columns that have achieved the balanced state along columns are assumed to be i_1, i_2, ..., i_k, respectively. Based on the balanced state definition, the maximum total load of nodes other than the first nodes in these k columns is bounded by $(n-1)[(i_1+1)+(i_2+1)+...(i_k+1)]$. Therefore, the total number is bounded by $I+(n-1)(I+k)+(n-k)(2n-1) \leq (2n-1)+(n-1)(2n+k-1)+(n-k)(2n-1)$ since $I = i_1+i_2+...+i_k \leq 2n-1$. Clearly, the total number is bounded by $4n^2-(2+k)n < 4n^2$. This number is maximized when $k=1$ and the corresponding distribution is shown in Figure 8. $\square$

With the above result, the extended SMART protocol can be resolved to the following steps:

- Step 1 (column-wise smoothing): Pre-processing on column (positive direction). If the last cluster fails condition 1 (discussed below), step 1 terminates, otherwise, simultaneous pre-

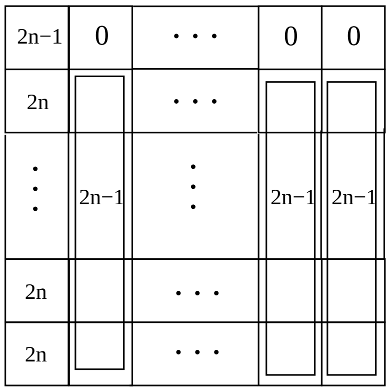


Figure 8: A worst case distribution.

processing and scan on column (negative direction). If the first cluster fails condition 2 (discussed below), step 1 terminates, otherwise, scan on column (positive direction).

- Step 2 (row-wise pre-processing and scan): Pre-processing on row (positive), followed by simultaneous pre-processing and scan on row (negative), finally scan on row (positive).
- Step 3 (column-wise scan): Scan on column (negative followed by positive).

Both conditions 1 and 2 are used for early termination when a particular column has less than $2n$ sensors. Condition 1 is defined as: the last cluster is included in a segment S and $W \geq 2C$. Condition 2 is defined as: the first cluster is included in a segment S such that $C = n$ and $W \geq 2n$. In step 1, each column needs 1, 2 or 3 sweeps depending on whether that column has $2n$ sensors or not. In step 2, 3 sweeps are needed and 2 sweeps are needed in step 3. In the worst case, 8 sweeps are needed.

The above approach has potential drawbacks in generating longer communication latency even in the absence of holes. To resolve this issue, we introduce some simple bookkeeping. Once the first sweep of step 1 is completed, each end node in the last row will set a flag to 1 whenever it registers at least $2n$ sensors in the corresponding segment. If all flags in the last row are set, step 3 can be skipped. Checking whether all flags are set can be done in parallel with step 2, which needs $2n$ steps with two sweeps on the last row. The first sweep is a scan using boolean AND and the second is a broadcast of the scan value of the first sweep which is a boolean value (1 for all flags set and 0 for otherwise).

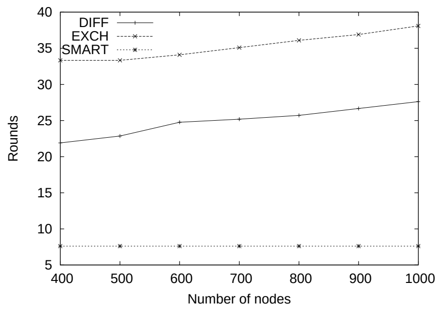
With the above modification, the worst case number of sweeps is reduced to 5. One more sweep can be eliminated by combining pre-processing and scan in step 1. Whenever the first cluster is included in the current segment, the scan process also starts. At the end of the first sweep, if the current segment includes both first and last clusters, the third sweep in step 1 can be eliminated since its function can be done at the second sweep. The optimization for number of moves discussed in Section III can still be used after the scan process starts. However, the number of moves during the smoothing and pre-processing phases cannot be further reduced.

6 Simulation

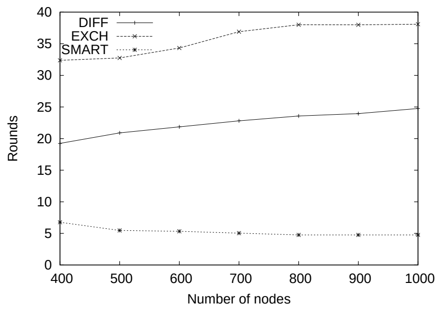
6.1 Simulation environment

We use a custom simulator. The initial deployment it generates could be a uniform or normal random distribution. We set up the simulation in a 500×500 area, which is the target field. The tunable parameters in our simulation are as follows. (1) Cluster numbers $n \times n$. Large n can improve the speed of deployment while small n can achieve more balanced results. We use 4 and 10 as n's values. (2) Number of sensors N. We have proved that at least $4n^2$ sensors are needed to guarantee the validation of SMART. Therefore, we vary N's value from 400 to 1000. We also include cases of under $4n^2$ sensors to check the robustness of SMART. (3) Normal distribution parameter σ. σ is the standard deviation of the normal distribution of the initial deployment, which can control the density degree of the sensor clustering. We use 1 to 5 as its values. When σ is 1, around 98% sensors are in 10% region of the area. When σ is 10, the distribution is very close to uniform random distribution. For each tunable parameter, the simulation is repeated 1000 times. In addition to the proposed algorithms, we also simulate the traditional load balancing algorithms diffusion (DIFF), dimension exchange (EXCH), and the Voronoi-based localized sensor redistribution algorithm (VOR) for comparison.

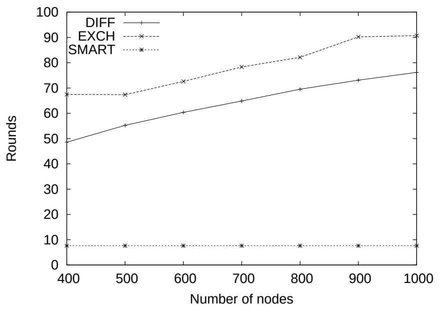
The performance metrics are (a) deployment quality and (b) cost. Deployment quality is shown by the balance degree measured by two simulation results. One is the *standard deviation* of the number of sensors in all the clusters. The other is *grads*, which is the difference between the largest cluster and the smallest one. Deployment cost is measured by the time of deployment, in terms of rounds, and energy consumption, in terms of overall moving distance.



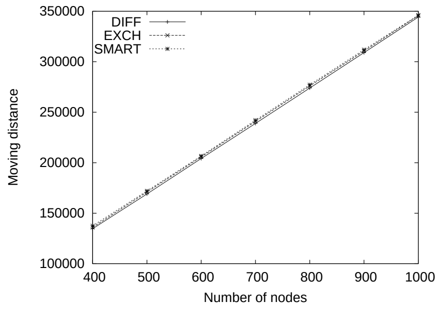
(a) $n = 4, \sigma = 1$



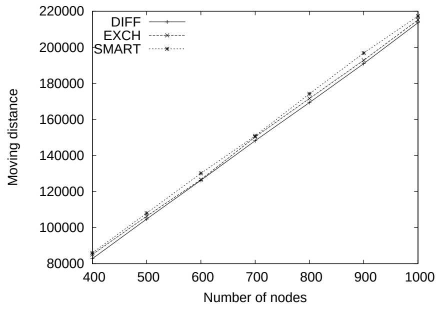
(b) $n = 4, \sigma = 5$



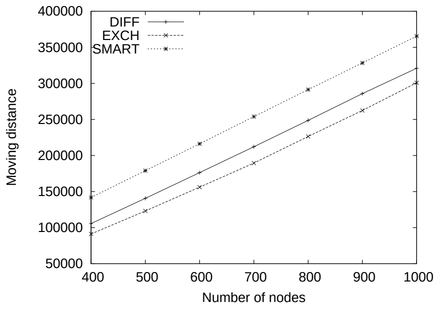
(c) $n = 10, \sigma = 1$



(d) $n = 4, \sigma = 1$



(e) $n = 4, \sigma = 5$



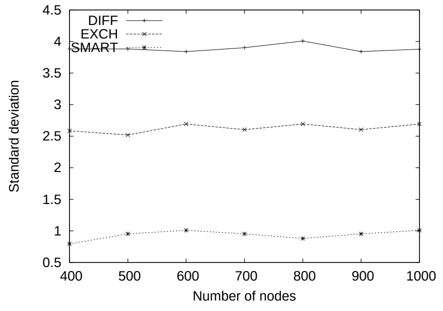
(f) $n = 10, \sigma = 1$

Figure 9: Comparison of DIFF, EXCH, and SMART in round number (a)-(c), and distance (d)-(f).

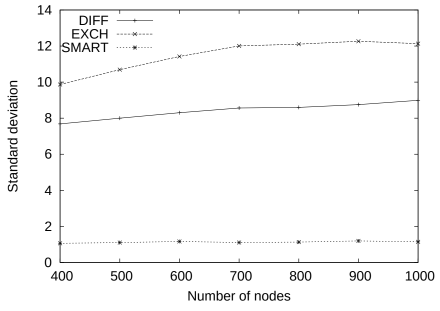
6.2 Simulation results

Figure 9 compares the number of rounds and moving distance of these three algorithms, DIFF, EXCH, and SMART in uniform random distribution. From (a)-(c) we can see that the proposed SMART has small and stable number of rounds. When the initial deployment is relatively balanced and n is small, every row could have more than $2n$ sensors, thus it has 5 rounds; otherwise, it takes 8 rounds (the worst case). Diffusion and dimension exchange both have large numbers of rounds, which increase with the growth of node number, especially when n is large and the initial deployment is uneven. (d)-(f) are the overall moving distance comparison. We can see that the overall sensor moving distance is proportional to the number of sensors. Therefore, average moving distance of a sensor is insensitive to node numbers in all these algorithms. Among the three, SMART has the largest moving distance. This is because it achieves the most balanced final state, which leads to more sensor movements.

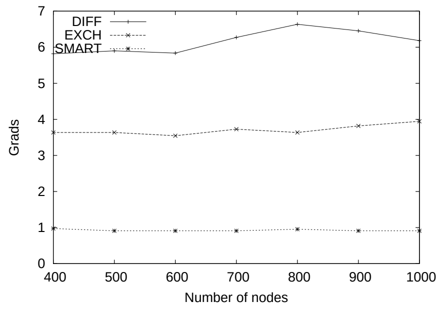
Figures 10 (a) and (b) show the balance degree of the results of these three algorithms by standard deviation in uniform random distribution. SMART achieves a balanced final state, and its standard



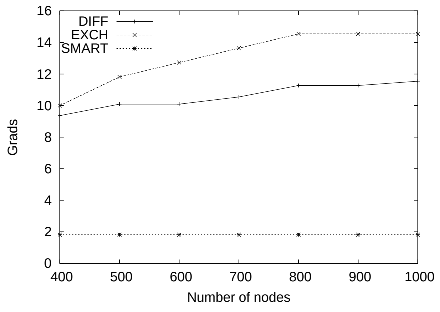
(a) Standard deviation ($n = 4$)



(b) Standard deviation, ($n = 10$)



(c) Grads, ($n = 4$)

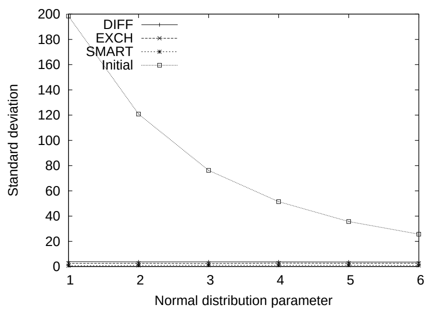


(d) Grads, ($n = 10$)

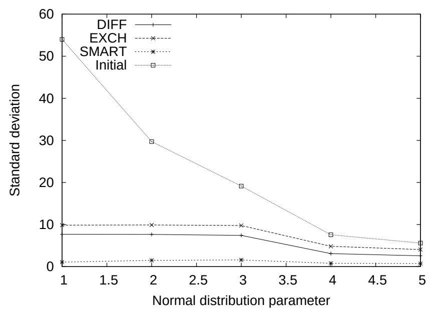
Figure 10: Balance degree of DIFF, EXCH, and SMART ($\sigma = 1$).

deviation is no more than 2. (c) and (d) are in terms of grads. The grads of SMART is no more than 2, and the grads in a row or a column is no more than 1. In DIFF and EXCH, only relative balanced state, the neighboring balance, is guaranteed. That is, the difference between adjacent clusters is no more than 1. Therefore, the result could be a ladder-like distribution, which leads to very large grads and standard deviation. When n is large, the grads of diffusion and dimension exchange are large, and their balance degrees are low.

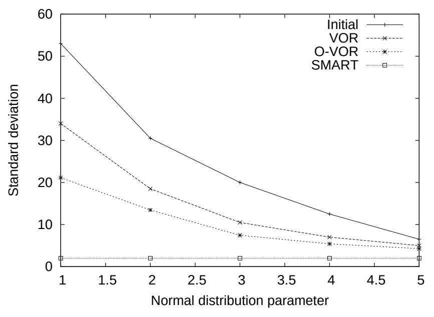
Figures 11 (a) (b) (d) and (e) compare the standard deviation and moving distance of algorithms using different normal distribution parameters σ. The curve ‘Initial’ is the standard deviation of the initial deployment. SMART can achieve a more balanced state than DIFF and EXCH. SMART also outperforms them in number of moves. In SMART, sensors move at most twice, one move for vertical direction and the other for horizontal; over 75% sensors move only once. When N is 400, and σ is 1, SMART has 444, diffusion has 1040, and dimension exchange has 1137. Since startup usually consumes more power than moving with invariable speed, less movement is desired. (c) is the standard deviation and (f) is moving distance comparison of VOR and SMART. We can see that VOR



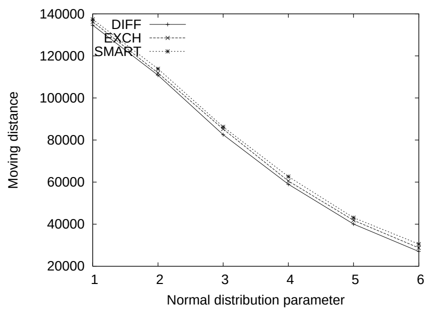
(a) Standard deviation, $n = 4$



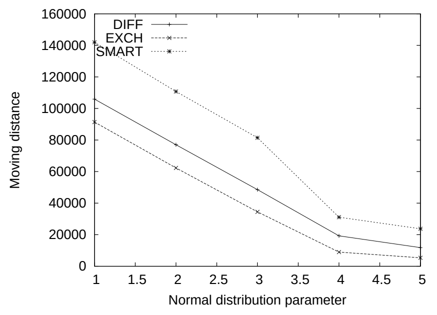
(b) Standard deviation, $n = 10$



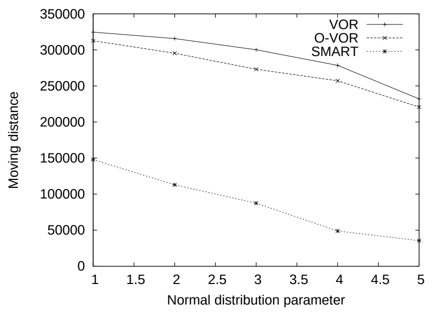
(c) Standard deviation, $n = 10$



(d) Moving distance, $n = 4$



(e) Moving distance, $n = 10$



(f) Moving distance, $n = 10$

Figure 11: SMART compares with DIFF, EXCH, and VOR using different σ ($N = 400$).

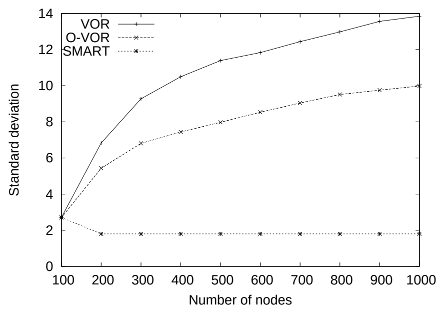
can only slightly reduce the standard deviation of initial deployment. It has been mentioned in [5] that the basic VOR algorithm has difficulties in dealing with high-degree clustering, where sensors are centered around a few locations. When σ is 1, after applying VOR, the clustering area still has high density, while the original blank area has low density. The optimized VOR (O-VOR) proposed to deal with this problem is better than VOR, but SMART still outperforms O-VOR.

VOR is designed for a relatively sparse sensor network that has a uniform random initial deployment, whereas SMART is designed for a relatively dense network with high-degree clustering. For fairness, we conduct the following simulation to compare the performance of SMART and VOR in a relatively sparse network where the condition of Theorem 7 for SMART is not necessarily satisfied.

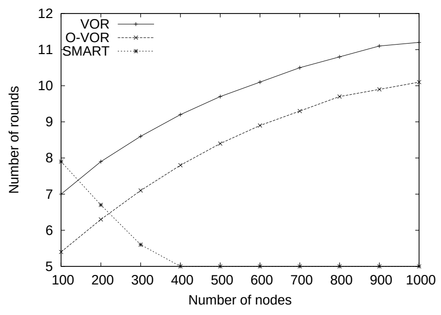
Figures 12 (a) and (b) show the comparisons of resultant balance degree (in terms of standard deviation) and number of rounds of SMART, VOR, and O-VOR ($\sigma = 3$, $n = 10$). In (a), when N is larger than 400, SMART guarantees the balanced final state, where the standard deviation of the resultant deployment of SMART should be less than 2. This result is consistent with the analytical results in the previous section, where if the average number of sensors in a cluster is less than 4, some rows may have less than $2n$ sensors after smoothing. When node number is smaller than 400, the standard deviation is larger than 2, and the balanced status is not achieved. However, the increase of standard deviation is small and the balance degree of SMART can still beat that of VOR. For VOR, when the node number is small, the resultant deployment is more balanced. With the growth of the number of deployed nodes, the balance degree gets lower. This is because in the high-degree clustering environment, when the coverage termination condition of VOR is met, most area can be covered by at least one node, but VOR terminates before nodes in the clustering area scatter out. (b) is the comparison of the number of rounds. At least 400 deployed nodes are needed to achieve the best performance, 5 rounds, for SMART. The worst is 8 rounds. For VOR, a smaller node number leads to fewer rounds. But VOR has fewer rounds than SMART when the node number is smaller than 150. O-VOR achieves more balanced degree with smaller round number than VOR.

Figures 12 (c) and (d) are the comparisons of the several variations of SMART, and also the optimal Hungarian based method (OPT) in uniform and normal random distributions, respectively ($n = 10, N = 500$). SMART(l), SMART(g), and SMART($m, 3$) are simulated. To check the effect of step (3) in threshold-based scan algorithm, we simulate SMART($g^{'}$), which is SMART(g) without step (3). In (c), SMART(m) has the most moving distance, while SMART(g) has a smaller moving distance than SMART(l). OPT has the smallest moving distance. (d) is results in normal random distribution. With the growth of σ, the moving distance decreases and the number of moves decreases slightly. SMART(g) and SMART(m) have smaller moving distances than SMART(l). SMART(m) has the smallest among the three. SMART(l) has close or even better performance than OPT because it does not achieve a balanced result as OPT does.

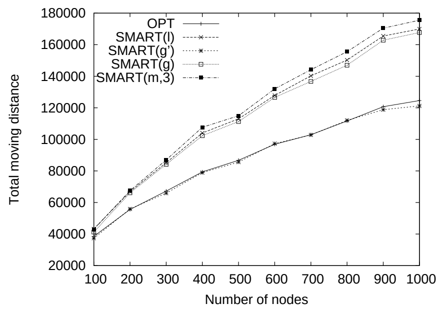
Simulation results can be summarized as follows: (1) SMART achieves a more balanced state than diffusion, dimension exchange, and Voronoi-based sensor deployment methods in unevenly deployed sensor networks. (2) SMART needs few rounds, which are bounded by 8, for load balancing. (3) The centralized optimal algorithm has the best performance; among all variations of SMART, SMART(g) has the best overall performance. (4) SMART can be effective when used in relatively dense sensor networks as a complement for the existing sensor deployment methods. (5) When number of deployed nodes is less than $4n^2$, the performance of SMART is reduced, since more rounds are needed and balanced final state cannot be achieved. (6) In sparse network, SMART may need more rounds than VOR to achieve a balanced degree, but it still beats VOR in terms of standard deviation.



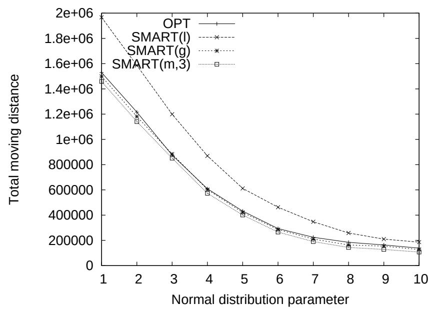
(a) Standard deviation



(b) Round number



(c) Moving distance, uniform



(d) Moving distance, normal

Figure 12: Property analysis of SMART and VOR; comparatione of variations of SMART.

7 Conclusion

In this paper, we have proposed a scan-based movement-assisted sensor deployment algorithm, which is a hybrid approach of local and global methods. We have considered a unique issue called communication hole, where certain sensing areas have no deployed sensors. A method of seed-planting has been proposed to move one senor to each uncovered area before the scanning process. We also develop an optimal solution which is the Hungarian method based. Simulation results show that the proposed method can achieve even deployment of sensors with modest costs. In the future, we will perform in depth simulation on energy consumption of sensor deployment algorithms and design some intra-cluster balancing algorithms to achieve high resolution load balancing. We also plan to consider the case where only parts of the sensors are mobile. In this case, the ultimate goal is to maximize the minimum load of these grids. This is a more general measurement for the balance degree of the final distribution.

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